



**Soil Sampling Data Analysis for Uranium: 235, 233/234, and 238 for areas U1 and U2**

**(Data collected May 8, 2010 and August 22, 2010)**



**By: Kristen Smith**

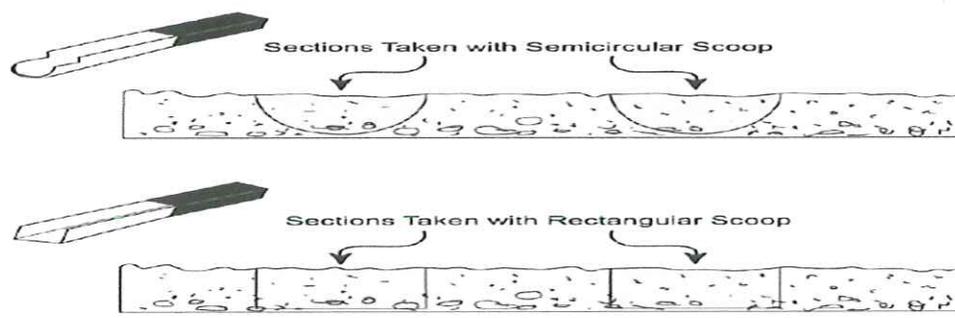
## Soil Sampling Techniques for Environmental Studies

This project's purpose is to compare three different methods of soil sampling for Hanford, including: Incremental Sampling Method (MIS), Judgmental Sampling, and Systematic Sampling. When this course is finished, a report will get sent to The Department of Ecology at Hanford with our findings. The site being analyzed is designated U1 and U2; this site is said to be contaminated. In May of 2010, samples were taken in these areas, then the contaminated soil was removed and the area was sampled once again in August of 2010. It's not really possible to collect and analyze the whole entire volume of soil for which decisions must be made; so, samples of that volume were collected for analysis to represent the whole population. The analysis on this type of data can be extremely high in cost and the number of samples collected is even inadequate sometimes because of their budget. Hanford is considering now, because of those limitations, the use of MIS. They want to make sure that the results are similar to the ones obtained from traditional systematic sampling methods.

**Incremental Sampling Methodology (MIS):** was developed to address some of the limitations mentioned above. MIS is a composite sampling approach where many (between 30 and 100) equal-mass increments are collected and combined in an unbiased manner from throughout the entire area of the soil/volume of interest. Once the combined increments are processed at the laboratory, a subsample is taken and analyzed.

**Judgmental Sampling:** is used when only a few discrete samples are collected. The number of samples collected is determined by negotiation, budget, professional judgment, or happenstance. The number of samples is often not based on statistical or other scientific rationale, and the location of the samples is often judgmental. Judgmental sampling plans are effective when source areas or migration pathways of high concentrations are being investigated.

**Systematic Sampling:** requires that the area of interest be divided up into a number of grids and then a random sample(s) is taken from each and analyzed from within each of the grids. The combined results are used to represent the area of concern. The number of samples for this type of analysis may be quite large.

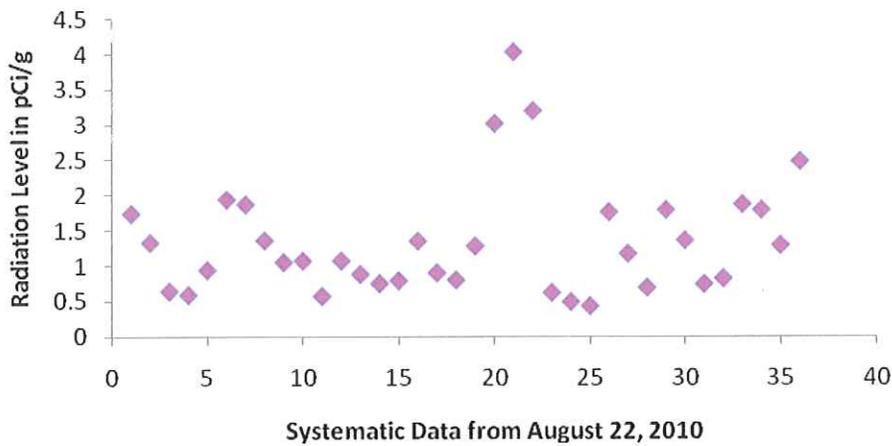


# Graphing and Summarizing Data

## 1. Graphic Investigation of U-238

Initially the scatter plot below of the U-238 systematic data for area U1 collected August 22, 2010 was constructed to investigate the radiation levels of this element that were present at the time the sample was collected. This plot also demonstrates the heterogeneity (or homogeneity) of this radioactive element in area U1.

### Uranium-238 Radiation Levels for U1

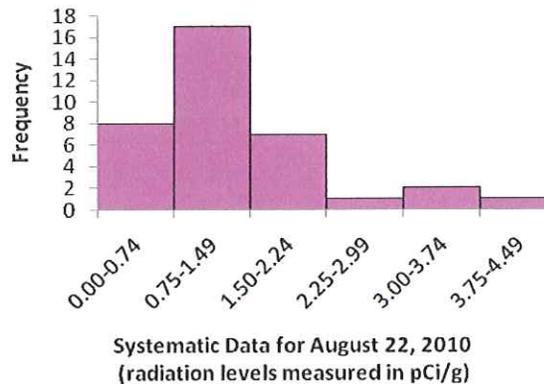


Further analysis of the isotope U-238 was conducted using the same systematic data from August 22, 2010. This analysis is summarized in the Frequency Table and Histogram presented below.

### Uranium 238 Radiation Levels for Area U1 Collected August 22, 2010

Radiation Level in (pCi/g)	Frequency
0.00-0.74	8
0.75-1.49	17
1.50-2.24	7
2.25-2.99	1
3.00-3.74	2
3.75-4.49	1

### Uranium 238 Levels for U1



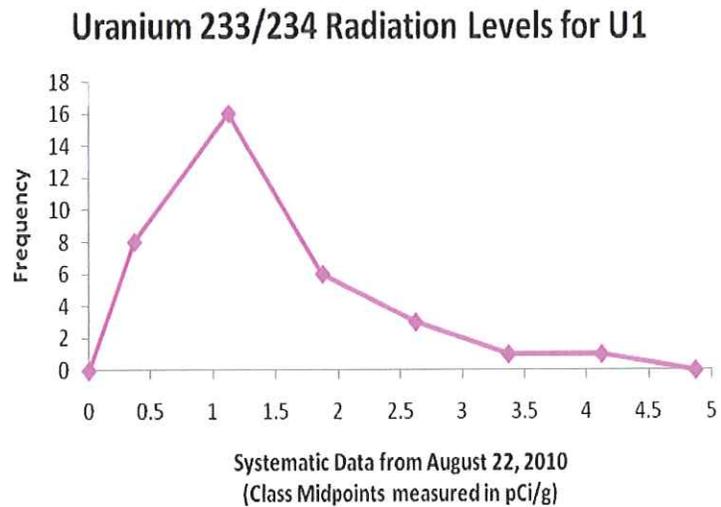
This analysis on the U-238 data from U1 August 22, 2010 was done by using Systematic Sampling. The results appear to be heterogeneous and also appear to be skewed to the right. The Histogram and Frequency Distribution support this claim. The analysis of this graph indicates that the radiation levels are primarily between 0.00 and 2.24 pCi/g. When looking at the scatter plot you can clearly see that there are four outliers, which are between 2.25 and 4.49 pCi/g.

## 2. Graphic Investigation of U-233/234

The Frequency Polygon pictured below was used to illustrate the radiation levels of the isotope U-233/234 from the systematic data for area U1 collected August 22, 2010. The graph was constructed using the class midpoints for the frequency distribution below. This graph also shows the nature of the distribution of this element in area U1 at the time it was collected.

**Uranium 238 Radiation Levels for Area U1 Collected August 22, 2010**

Radiation Level in pCi/g	Frequency
0.00-0.74	8
0.75-1.49	16
1.50-2.24	7
2.25-2.99	3
3.00-3.74	1
3.75-4.49	1



The chart and graph above highlights the low radiation levels (between 0.00 and 2.24 pCi/g) of the Uranium 233/234 Systematic data that was collected in area U1 on August 22, 2010. There are only five samples with, higher radiation levels. All five values are between 2.25 and 4.49 pCi/g.

### 3. Comparison of U-238 and U-233/234

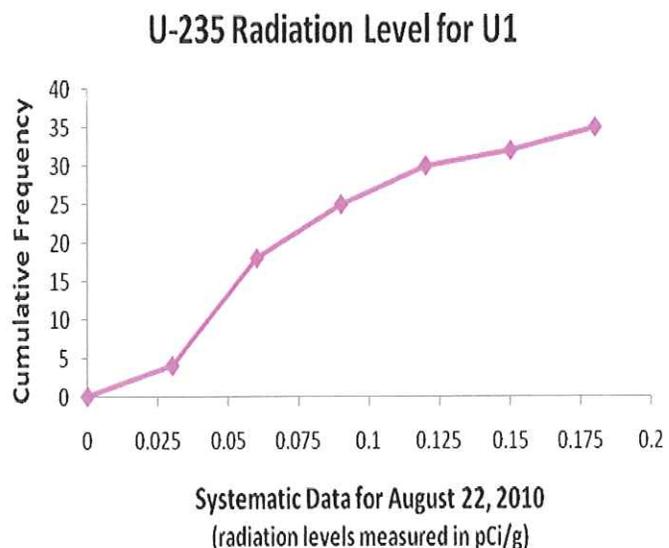
Comparing all of the above charts, it appears that the radiation levels don't differ by too much. The majority, stay in the range from 0.75-1.49 pCi/g, which shows that the radiation levels from the Uranium 238 and the Uranium 233/234 are pretty low. Both charts (histogram and the frequency polygon) are skewed to the right, which indicates that very few had radiation levels between 2.25-4.49 pCi/g.

### 4. Graphic Investigation of U-235

The radioactive isotope U-235 was analyzed using a Cumulative Frequency Distribution in conjunction with an Ogive (cumulative frequency graph). The systematic data for this isotope was collected from area U1 on August 22, 2010. The single sample, B242080, had a level of 1.34 pCi/g and was left out of this analysis. This value is considered an outlier and is assumed to be unrepresentative of this area.

Uranium 238 Radiation Levels for Area U1 Collected August 22, 2010

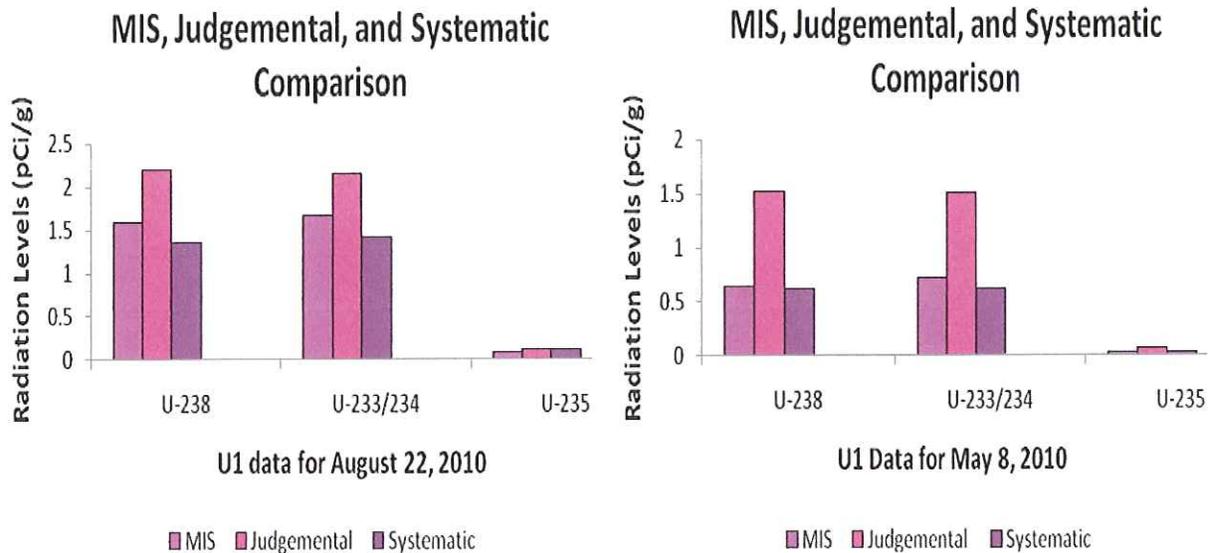
Radiation Level in pCi/g	Cumulative Frequency
Less than 0.030	4
Less than 0.060	18
Less than 0.090	25
Less than 0.120	30
Less than 0.150	32
Less than 0.180	35



The above chart and graph, for the U-235 systematic data, which was collected from area U1 on August 22, 2010, are helpful in showing the number of samples that are below a certain radiation level. There were a total of 36 samples in this specific data set, and only 35 were used on the above chart and graph. One sample was left out (B242080) because its radiation level was 1.34 pCi/g, which is much higher than any of the others, and is considered unusual.

## 5. Graphic Comparison of Sampling Methods for U-238, U-233/234, and U-235

A comparison analysis was conducted on the levels of radiation in the MIS, Judgmental, and systematic data collected from area U1 for U-238, U-233/234, and U-235 on August 22, 2010 and on May 8, 2010. The Multiple Bar graphs below were used to analyze this information.



In comparing the MIS and Systematic data, it seems that they both produce relatively the same results. With this being said, Hanford should consider switching to MIS sampling. In the long run, it would be more cost effective than the current Systematic sampling; however, more testing would be needed to verify this. When looking at all three sampling methods together, you can clearly see that Judgmental sampling shows a much higher radiation level.

## 6. Graphic Investigation of Chromium

A Stem and Leaf Plot was used to investigate the levels of Chromium contained in the systematic data for area U1 collected on August 22, 2010. The plot was constructed with the stems representing the ones digit of the original data values and the leaves representing the tenths digit of the original data values. This plot not only shows the nature of the Chromium distribution in the area U1 at the time of collection but allows for the preservation of the original data values.

### Chromium for Area U1 in mg/kg

Stem (Ones)	Leaf (Tenths)
4	9
5	5 5 6 6 7 8 8
6	1 2 4 4 4 7 8 9
7	0 2 4 4 5 5 6 8
8	0 5 5 5 7 8
9	0 2 2 5 9
10	0

The Chromium Stem and Leaf Plot uses the Systematic Data collected from U1 on August 22, 2010 and shows a normal distribution. Rotating this table about  $90^\circ$ , results in a noticeable bell-shaped curve around the numbers that are in the leaves. This plot is referring to the original data values.

# Descriptive Statistics and Probability

## 1. Descriptive Statistics for U-238, U2344/234, and U-235

The first part of this report analyzed data collected primarily from area U1 on August 22, 2010. This section of the report will analyze data collected primarily from area U1 on May 8, 2010. Measures of central tendency and variation are extremely important when analyzing a data set. Thus, the following calculations were made and summarized in the table below for the isotopes U-238, U-233/234, and U-235 using the systematic data for U1 collected on May 8, 2020. Results were rounded to three decimal places.

### Summary Statistics for the Systematic Data Done on May 8, 2010 from Area U1

pCi/g	U-238	U-233/234	U-235
Mean	0.624	0.617	0.0327
Median	0.56	0.555	0.032
Standard Deviation	0.239	0.206	0.0164
Range	1.18	1.02	0.08

The above information was used to calculate the minimum and maximum “usual” values for each isotope. Two techniques were used for these calculations, the Empirical Rule for 95% of data values and Chebyshev’s Theorem for at least 93.75% of data values.

The Empirical Rule assumes that the population is normally distributed. It states that 95% of data values fall within 2 standard deviations of the mean. These calculations are presented below.

#### The 95% Empirical Rule for U-238:

$$\begin{aligned}\text{Minimum Usual Radiation Level in pCi/g: } \bar{x} - 2s &= 0.624 - 2(0.239) \\ &= 0.146 \text{ pCi/g}\end{aligned}$$

$$\begin{aligned}\text{Maximum Usual Radiation Level in pCi/g: } \bar{x} + 2s &= 0.624 + 2(0.239) \\ &= 1.102 \text{ pCi/g}\end{aligned}$$

**The 95% Empirical Rule for U-233/234:**

$$\begin{aligned}\text{Minimum Usual Radiation Level in pCi/g: } \bar{x} - 2s &= 0.617 - 2(0.206) \\ &= 0.205 \text{ pCi/g}\end{aligned}$$

$$\begin{aligned}\text{Maximum Usual Radiation Level in pCi/g: } \bar{x} + 2s &= 0.617 + 2(0.206) \\ &= 1.029 \text{ pCi/g}\end{aligned}$$

**The 95% Empirical Rule for U-235:**

$$\begin{aligned}\text{Minimum Usual Radiation Level in pCi/g: } \bar{x} - 2s &= 0.0327 - 2(0.0164) \\ &= -0.0001 \text{ pCi/g}\end{aligned}$$

$$\begin{aligned}\text{Maximum Usual Radiation Level in pCi/g: } \bar{x} + 2s &= 0.0327 + 2(0.0164) \\ &= 0.0655 \text{ pCi/g}\end{aligned}$$

Chebyshev's Theorem makes no assumptions about the distribution of the population from which the data was sampled. As a result, it provides a conservative estimate of the minimum and maximum "usual" values of a data set. This theorem states that at least 93.75% of data values fall within 4 standard deviations of the mean. These calculations are presented below.

**The 93.75% Chebyshev's Theorem for U-238:**

$$\begin{aligned}\text{Minimum Usual Radiation Level in pCi/g: } \bar{x} - 4s &= 0.624 - 4(0.239) \\ &= -0.332 \text{ pCi/g}\end{aligned}$$

$$\begin{aligned}\text{Maximum Usual Radiation Level in pCi/g: } \bar{x} + 4s &= 0.624 + 4(0.239) \\ &= 1.58 \text{ pCi/g}\end{aligned}$$

**The 93.75% Chebyshev's Theorem for U-233/234:**

$$\text{Minimum Usual Radiation Level in pCi/g: } \bar{x} - 4s = 0.617 - 4(0.206)$$

$$= -0.207 \text{ pCi/g}$$

$$\text{Maximum Usual Radiation Level in pCi/g: } \bar{x} + 4s = 0.617 + 4(0.206)$$

$$= 1.441 \text{ pCi/g}$$

**The 93.75% Chebyshev's Theorem for U-235:**

$$\text{Minimum Usual Radiation Level in pCi/g: } \bar{x} - 4s = 0.0327 - 4(0.0164)$$

$$= -0.0329 \text{ pCi/g}$$

$$\text{Maximum Usual Radiation Level in pCi/g: } \bar{x} + 4s = 0.0327 + 4(0.0164)$$

$$= 0.0983 \text{ pCi/g}$$

According to the 95% Empirical Rule and the 93.75% Chebyshev's Theorem this data is most likely to be between the minimum and maximum values. The above information states the "usual" values for the Systematic data on May 8, 2010. Below, states the "unusual" values for both the Empirical Rule and Chebyshev's Theorem. Those values were the ones that didn't fit into the above category, as "usual," and they were all a part of the same data.

**Identification of the Unusual Values**

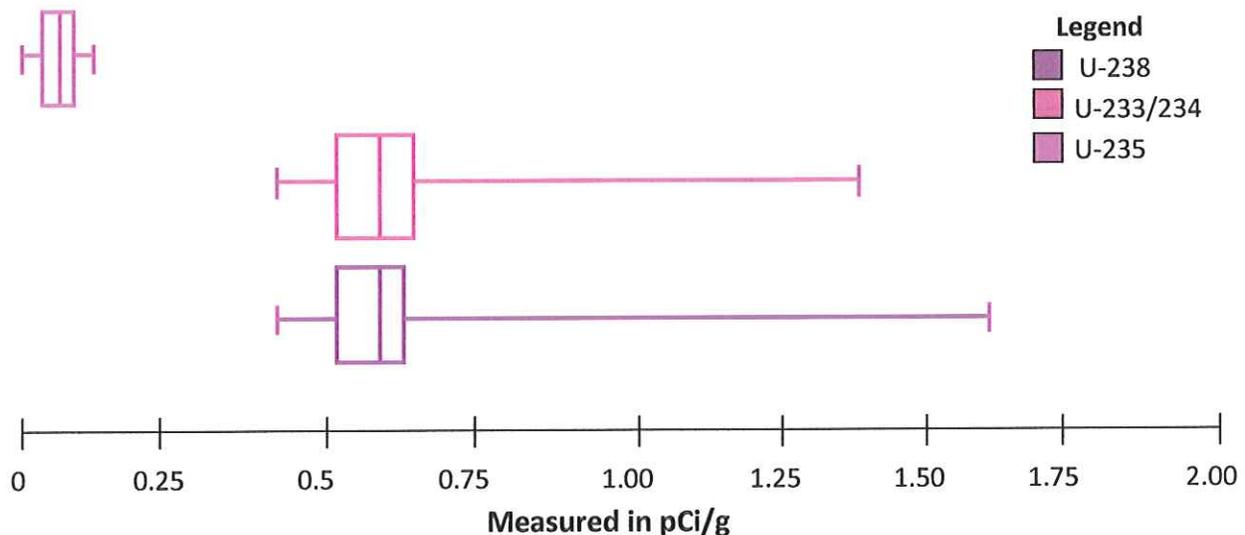
	95% Empirical Rule	Chebyshev's Theorem
U-238	1.13	1.62
	1.33	
	1.62	
U-233/234	1.03	
	1.04	
	1.19	
	1.43	
U-235	0.066	
	0.086	

The Chebyshev's Theorem is the one that should be used for this specific data because it can be used for any data set, as opposed to the Empirical Rule which is only used for data with a normal distribution that this doesn't have. The information set up in the box plots below, clearly is skewed to the right.

Descriptive statistical analysis also includes a 5-number consisting of the minimum value, the 25th percentile, the median, the 75th percentile, and the maximum value. These values are used to construct a boxplot representing the data set of interest. In this analysis 5-number summaries and stacked boxplots were calculated for the systematic data collected on May 8, 2010 from area U1 for U-238, U-233/234, and U-235. This information is presented below.

pCi/g	U-238	U-233/234	U-235
Minimum Value	0.44	0.41	0.006
25 <sup>th</sup> percentile	0.5	0.53	0.021
Median	0.56	0.555	0.032
75 <sup>th</sup> percentile	0.625	0.64	0.0415
Maximum Value	1.62	1.43	0.086

**Comparison Boxplots of Uranium Systematic Data Collected from U1 on May 8, 2010**



The boxplot shown above gives the differences between all three sample sets and their radiation levels that lie within the systematic data for Uranium 238, 233/234, and 235. The U-235 is the sample with very little radiation. The other two, the U-233/234 and U-238, are very

similar in their results. They seem to have a much higher radiation level in pCi/g, than that of the U-235. All three of these distributions are skewed to the right.

## 2. Selecting Subsamples of U-238

Researchers observed several anomalies in the U-238 systematic data collect from area U1 on May 8, 2010. As a result of these observations, they were tasked with performing a more in depth analysis of this data. Due to budget constraints only 10 of the original samples will be selected for this extended analysis. The number of different subsamples of size 10 was determined using the following calculation:

$$\begin{aligned} {}_{40}C_{10} &= \frac{40!}{(40-10)!10!} \\ &= 847,660,528 \text{ different ways} \end{aligned}$$

The above calculation of 847,660,528, represents the number of different ways that the researchers can randomly select 10 subsamples from the original 40 samples in the U-238 data set on May 8, 2010.

Once the researchers selected their 10 subsamples for extended analysis they were instructed to initially test only 4 of these subsamples. The order in which these 4 subsamples are tested is critical to their in-depth analysis. The number of different orders that 4 of the 10 subsamples can be selected and tested was determined using the following calculation:

$$\begin{aligned} {}_{10}P_4 &= \frac{10!}{(10-4)!} \\ &= 5,040 \text{ different orders} \end{aligned}$$

The calculations above represent the 5,040 different orders in which 4 of 10 subsamples can be selected from the U-238 systematic data from May 8, 2010.

### 3. Sample Probabilities for U-238, U-233/234, and U-235

Part of this soil sample research involves calculating different probabilities. The table below presents 120 different measurements taken of the radiation levels for U-238, U-233/234, and U-235 for the systematic data from U1 collected on May 8, 2010. The information in this table was used to calculate these probabilities.

**Radiation Levels for the Systematic Data Collect on May 8, 2010 in Area U1**

pCi/g	U-238	U-233/234	U-235	Totals
0.00-0.50	10	8	40	58
0.51-1.00	26	28	0	54
1.01-1.51	3	4	0	7
1.51-2.00	1	0	0	1
<b>Totals</b>	40	40	40	120

A single measurement was selected at random for analysis. The probability that the measurement had a radiation level of 0.50 pCi/g or less where the source of the radiation was unknown was calculated as follows:

$$\begin{aligned} P\left(\text{radiation level} \leq 0.50 \frac{\text{pCi}}{\text{g}}\right) &= \frac{\text{Total number of radiation levels} \leq 0.50}{\text{Total Number of Measurements}} \\ &= \frac{58}{120} \\ &= 0.483 \end{aligned}$$

Two samples of size 4 each were selected at random and only their U-238 radiation level was analyzed. The probability that all 4 samples would have a radiation level of 0.50 pCi/g or less was evaluated. The first sample was selected with replacement. The second sample was selected without replacement. The detailed calculations of each of these probabilities are presented below.

**With Replacement:**

$$\begin{aligned} P(\text{All 4 U238 samples} \leq 0.50 \text{ pCi/g}) &= \frac{10}{40} \times \frac{10}{40} \times \frac{10}{40} \times \frac{10}{40} \\ &= 0.00391 \end{aligned}$$

**Without Replacement:**

$$\begin{aligned} P(\text{All 4 U238 samples} \leq 0.50 \text{ pCi/g}) &= \frac{10}{40} \times \frac{9}{39} \times \frac{8}{38} \times \frac{7}{37} \\ &= 0.00230 \end{aligned}$$

A sample size of 4 was selected at random (with replacement) in which only the U-233/234 radiation levels were evaluated. To determine the probability that at least 1 out of these 4 samples has a radiation level that is greater than 0.50 pCi/g the following calculations were done.

**With Replacement:**

$$\begin{aligned} P(\text{All 4 U238 samples} \leq 0.50 \text{ pCi/g}) &= \frac{8}{40} \times \frac{8}{40} \times \frac{8}{40} \times \frac{8}{40} \\ &= 0.0016 \end{aligned}$$

$$\begin{aligned} P(\text{At least one has radiation} > 0.50 \text{ pCi/g}) &= 1 - P(\text{none with radiation} > 0.50 \text{ pCi/g}) \\ &= 1 - P(\text{All 4 radiation levels} \leq 0.50 \text{ pCi/g}) \\ &= 1 - 0.0016 \\ &= 0.998 \end{aligned}$$

Two measurements were to be randomly selected, without replacement, from the 120 radiation level measurements. The probability that the first would have a radiation level greater than 1.00 pCi/g source unknown and second would have a radiation level of 0.50 pCi/g or less source unknown was calculated as shown below.

$$\begin{aligned}
 P(1^{\text{st}} > 1.00 \text{ pCi/g and } 2^{\text{nd}} \leq 0.50 \text{ pCi/g}) &= P(1^{\text{st}} > 1.00 \text{ pCi/g})P(2^{\text{nd}} \leq 0.50 \text{ pCi/g} | 1^{\text{st}} > 1.00 \text{ pCi/g}) \\
 &= \frac{7 + 1}{120} \times \frac{58}{119} \\
 &= \frac{8}{120} \times \frac{58}{119} \\
 &= 0.0325
 \end{aligned}$$

A sample was found in the lab after all the other samples had been properly stored. This sample had a radiation level between 0.00 and 0.50 pCi/g. The researchers decided to use probability to help them find the source of this radiation. The following probabilities were calculated to assist them in this determination.

$$\begin{aligned}
 P(\text{U238 source} | \text{radiation level between } 0.00 \text{ and } 0.50 \text{ pCi/g}) &= \frac{\# \text{ of U238 values between } 0.00\text{-}0.50 \text{ pCi/g}}{\text{total } \# \text{ of values between } 0.00\text{-}0.50 \text{ pCi/g}} \\
 &= \frac{10}{58} \\
 &= 0.172
 \end{aligned}$$

$$\begin{aligned}
 P(\text{U233/234} | \text{radiation level between } 0.00\text{-}0.50 \text{ pCi/g}) &= \frac{\# \text{ of U233/234 values between } 0.00\text{-}0.50 \text{ pCi/g}}{\text{total } \# \text{ of values between } 0.00\text{-}0.50 \text{ pCi/g}} \\
 &= \frac{8}{58} \\
 &= 0.138
 \end{aligned}$$

$$\begin{aligned}
 P(\text{U235 source} | \text{radiation level between } 0.00\text{-}0.50 \text{ pCi/g}) &= \frac{\# \text{ of U235 values between } 0.00\text{-}0.50 \text{ pCi/g}}{\text{total } \# \text{ of values between } 0.00\text{-}0.50 \text{ pCi/g}} \\
 &= \frac{40}{58} \\
 &= 0.690
 \end{aligned}$$

The misplaced sample, which was left behind after all of the other samples had been stored, was most likely U-235; this is noticeable, mainly, because the probability of 0.690 pCi/g is quite high, as opposed to the other two samples.

# Analysis Samples and Probability Distributions

## 1. Ensuring Enough Samples for Analysis

The Hanford test planning team determined that the minimum number of samples required for analysis of their soil data would be 35. The test engineer is aware that samples can be excluded from the analysis for a variety of legitimate reasons. To ensure that at least 35 of the samples are fit for analysis, the test engineer proposes that they collect 40 samples. He uses the following Probability Distribution of usable data samples from a sample of size 40 to support his proposal.

### Probability Distribution of Usable Samples from a Sample of Size 40

X	P(x)	xP(x)	x <sup>2</sup> P(x)
40	0.021	0.84	33.6
39	0.153	5.967	232.713
38	0.281	10.678	405.764
37	0.253	9.361	346.357
36	0.139	5.004	180.144
35	0.104	3.64	127.4
34	0.023	0.782	26.588
33	0.015	0.495	16.335
32	0.011	0.352	11.264
<b>Totals</b>	<b>1.000</b>	<b>37.119</b>	<b>1380.165</b>

According to this Probability Distribution of Usable Samples, the expected number (mean) of usable samples out of 40 samples, would be  $\mu = \sum X_i P(X_i)$ , which is 37.1 samples. In addition the standard deviation for this distribution was calculated using the following formula,

$\sigma = \sqrt{\sum X_i^2 P(X_i) - [\sum X_i P(X_i)]^2}$ . This calculation resulted in a standard deviation of 1.5 samples. With a mean of 37.1 samples and a standard deviation of 1.5 samples, the following formulas were used to find the minimum and maximum usual values:

#### Minimum and Maximum Standard Deviation Formula:

$$\text{Minimum Usual Value: } \mu - 2\sigma = 37.1 - 2(1.5) = 34.1 \text{ samples}$$

$$\text{Maximum Usual Value: } \mu + 2\sigma = 37.1 + 2(1.5) = 40.1 \text{ samples}$$

The minimum usual number of usable samples out of 40, is 34.1 samples. The maximum usual number of usable samples out of 40, is 40.1 samples. The test engineer can feel quite comfortable with his choice of 40 samples. By following this approach, he will have the correct number of usable data samples (at least 35) approximately 95% of the time.

## 2. Results Validation and Quality Control

Critical decisions about whether or not to cleanup areas U1 and U2 are to be made as a result of this analysis. To ensure the validity and quality of these results the test analysts require data from at least at 13 duplicate tests. The test engineer is aware of the fact that any given test may fail to meet the minimum usable sample requirement of at least 35 usable samples. To ensure that 13 usable duplicate tests are collected, he proposes that they conduct 15 duplicate tests. To support his proposal, initially the test engineer calculated the probability of a given sample having at least 35 usable samples out of 40 collected samples.

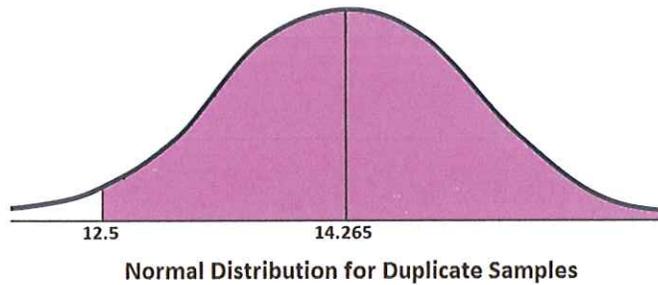
$$\begin{aligned}P(\text{at least 35 out of 40 usable samples}) &= 1 - P(34) - P(33) - P(32) \\ &= 1 - 0.023 - 0.015 - 0.01 \\ &= 0.951\end{aligned}$$

He then proceeded to calculate the probability of at least 13 usable tests out of 15 tests conducted using two different techniques. The first technique that he used involved the Binomial Probability Formula and the second technique that he used was the Normal Approximation to the Binomial Formula. These calculations are presented below.

### Binomial Probability Formula:

$$\begin{aligned}P(\text{at least 13 out of 15}) &= {}_{15}C_{13}(0.951)^{13}(0.049)^2 + {}_{15}C_{14}(0.951)^{14}(0.049)^1 + {}_{15}C_{15}(0.951)^{15}(0.049)^0 \\ &= 0.131198291 + 0.363759694 + 0.470660502 \\ &= 0.965618487 \\ &= 0.966\end{aligned}$$

**Normal Approximation:**



$$\begin{aligned}
 P(\text{at least 13 out of 15}) &= P(x \geq 13) \\
 &= P(x > 12.5) \\
 &= 1 - P(x < 12.5) \\
 &= 1 - P\left(z < \frac{12.5 - 14.265}{0.836053228}\right) \\
 &= 1 - 0.0174 \\
 &= 0.9826 \\
 &= 0.983
 \end{aligned}$$

The test engineer should feel quite comfortable with his choice of conducting 15 duplicate tests. According to his calculations, he will have the correct number of duplicate tests approximately 96.6% of the time. In this case, it is inappropriate to use the Normal Approximation to the Binomial Formula calculations above. This particular data set fails the requirement that  $nq \geq 5$  since  $nq = 15(0.049)$  which is 0.735.

**3. Probabilities Involving Uranium Radiation Levels**

The Comprehensive Environmental Response, Compensation, and Liability Act (CERCLA) requires the cleanup of surface soil having a radiation level of 5 pCi/g or more. The table below lists the samples that exceed that level.

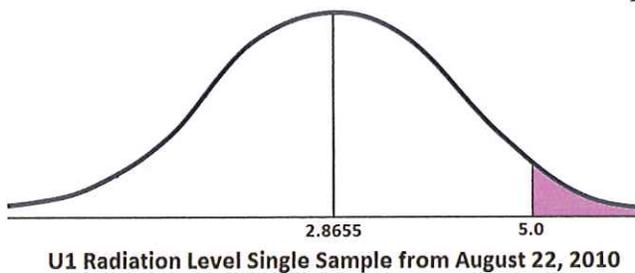
**Samples with Radiation Level Above 5 pCi/g**

Uranium Levels U1		Uranium Levels U2	
May 8, 2010	August 22, 2010	May 8, 2010	August 22, 2010
None	B24208 B24209 B24210 B24226	None	B24260 B24260

Out of the 40 samples collected from areas U1 and U2 on May 8, 2010, none were above the radiation level of 5 pCi/g. However, of the 36 samples collected from area U1 on August 22, 2010, there were 4 samples with radiation levels that exceeded 5 pCi/g. During the same time period, of the 39 samples collected from area U2, there were 2 samples with radiation levels greater than 5 pCi/g. These results indicate that further analysis, and perhaps cleanup, should be done prior to considering this area clean.

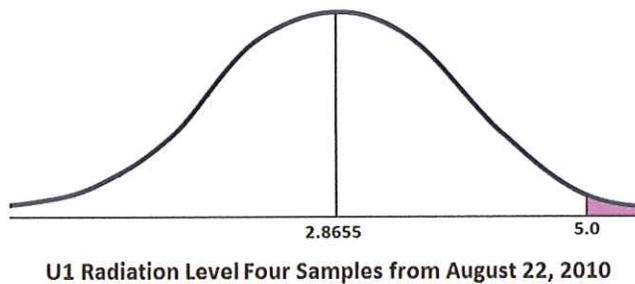
As a follow up to the reporting of sample radiation it has been determined that the August 22, 2010 sample data needs to be analyzed in greater detail. Due to budget constraints not all these samples can be analyzed again. Therefore the following probabilities were calculated to determine the probability of selecting one sample with a radiation level that may result in cleanup of the area U1. The probability was also calculated of selecting four samples with a mean radiation level that may result in the cleanup of area U1.

### Single Sample:



$$\begin{aligned}
 P(x > 5.0 \text{ pCi/g}) &= 1 - P(x < 5.0) \\
 &= 1 - P\left(z < \frac{5.0 - 2.8655}{1.7336}\right) \\
 &= 1 - P(z < 1.23) \\
 &= 1 - 0.8907 \\
 &= 0.109
 \end{aligned}$$

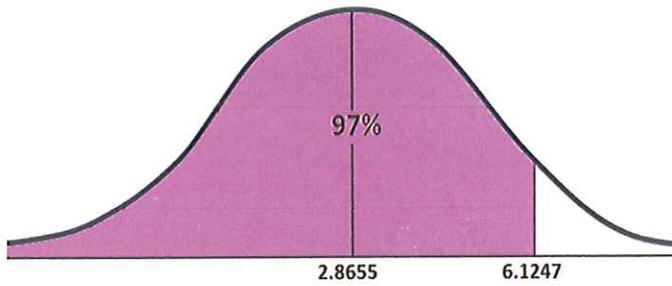
### Four Samples:



$$\begin{aligned}
 P(\bar{x} > 5.0 \text{ pCi/g}) &= 1 - P(x < 5.0) \\
 &= 1 - P\left(z < \frac{5.0 - 2.8655}{1.7336/\sqrt{4}}\right) \\
 &= 1 - P(z < 2.46) \\
 &= 1 - 0.9931 \\
 &= 0.007
 \end{aligned}$$

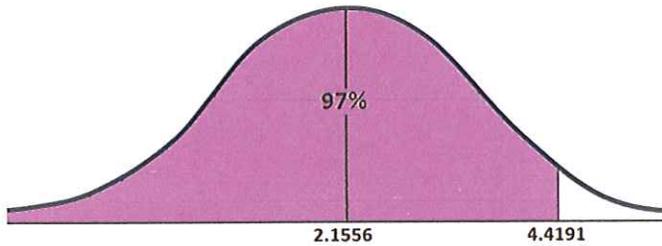
## 4. Percentiles Involving Uranium Radiation Levels

The assumption of normality was made with respect to the radiation levels of Uranium in areas U1 and U2 collected on August 22, 2010 in order to calculate the ninety-seventh percentile. This value represents the radiation level that separates the bottom 97% of radiation levels from the top 3%. A decision to declare an area clean will be made if at least 97% of the data values fall below 5 pCi/g. Otherwise, further testing will be conducted to determine if cleanup is required.



U1 97th Percentile for August 22, 2010

$$\begin{aligned}
 x &= \bar{x} + 1.88s \\
 &= 2.8655 + 1.88(1.7336) \\
 &= 6.124668 \\
 &= 6.1247 \text{ pCi/g}
 \end{aligned}$$



U2 97th Percentile for August 22, 2010

$$\begin{aligned}
 x &= \bar{x} + 1.88s \\
 &= 2.1556 + 1.88(1.204) \\
 &= 4.4191 \text{ pCi/g}
 \end{aligned}$$

The value 6.1247 pCi/g represents the 97<sup>th</sup> percentile for area U1. This value is larger than the 5 pCi/g level and, thus, marks this area for further study and possible cleanup. On the other hand, the value of 4.4191 pCi/g representing the 97<sup>th</sup> percentile for area U2, is below the 5 pCi/g level. This area can now be declared clean and no further study is required.

## Confidence Intervals and Hypothesis Tests

In 2008, the Washington State Department of Ecology in cooperation with the Department of Energy (DOE) and the CH2M HILL Plateau Remediation Company (CHPRC) collected two samples from a single location immediately adjacent to the pipe discharge point for the pond designated 216-S-19. These samples were designated as "screening" samples in order to determine if the site would be a suitable location to conduct a comparison of three sampling designs (Judgmental, MIS, and Systematic). Based on the results of this screening effort, contaminants of potential concern (COPCs) were identified. The contaminants were Nitrate, Copper, Chromium, Zinc, Uranium-233/234, Uranium-238, Uranium-235, Plutonium-239/240, Plutonium-238, and Americium-241. This section of the report will be looking at the total radiation levels of Uranium and Nitrate.

### 1. Confidence Intervals on the Proportion of the Pond with High Radiation Levels of Uranium

DOE is concerned about the proportion of this pond that contains Uranium radiation levels above the CERCLA standard of 5 pCi/g. The Systematic sample data collected on August 22, 2010 from both areas U1 and U2 was used to address this concern. For the proportion of the contaminant in each area, a 90% confidence interval was constructed. It was assumed that all requirements for constructing this type of confidence interval were satisfied.

**Area U1:** The best point estimates for the proportion of the pond with Uranium radiation levels greater than 5 pCi/g for area U1 is calculated below:

$$\hat{p} = \frac{\text{number of U1 samples with Uranium radiation level} > 5 \text{ pCi/g}}{\text{total number of U1 Uranium samples}}$$

$$\hat{p} = \frac{4}{36}$$

$$\hat{p} = \frac{1}{9} \quad (\hat{p} \approx 0.111)$$

Given a 90% confidence interval, the margin of error (E) was calculated using = 0.10 significance interval.

$$\text{given: } Z_{\alpha/2} = Z_{0.10/2} = Z_{0.05} = 1.645 \text{ and: } \hat{q} = 1 - \frac{1}{9} = \frac{8}{9}$$

$$E = Z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$E = 1.645 \sqrt{\frac{\left(\frac{1}{9}\right)\left(\frac{8}{9}\right)}{36}}$$

$$= 0.086162271$$

$$= 0.086$$

The information from above was used to calculate 90% Confidence Intervals for the proportions of the pond that have Uranium Radiation levels greater than 5 pCi/g.

$$\hat{p} - E < p < \hat{p} + E$$

$$0.111 - 0.0862 < p < 0.111 + 0.0862$$

$$0.025 < p < 0.197$$

**Area U2:** The best point estimates for the proportion of the pond with Uranium radiation levels greater the 5pCi/g for area U1 are calculated below:

$$\hat{p} = \frac{\text{number of U2 samples with Uranium radiation levels} > 5 \text{ pCi/g}}{\text{total number of U2 Uranium samples}}$$

$$\hat{p} = \frac{2}{39} \quad (\hat{p} \approx 0.0513)$$

Given a 90% confidence interval, the margin of error (E) was calculated using = 0.10 significance interval.

$$\text{given: } Z_{\alpha/2} = Z_{0.10/2} = Z_{0.05} = 1.645 \text{ and: } \hat{q} = 1 - \frac{2}{39} = \frac{37}{39}$$

$$E = Z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$E = 1.645 \sqrt{\frac{\left(\frac{2}{39}\right)\left(\frac{37}{39}\right)}{39}}$$

$$= 0.058101166$$

$$= 0.0581$$

The information from above was used to calculate 90% Confidence Intervals for the proportions of the pond that have Uranium Radiation levels greater than 5 pCi/g.

$$\begin{aligned}\hat{p} - E &< p < \hat{p} + E \\ 0.051 - 0.058 &< p < 0.051 + 0.058 \\ -0.007 &< p < 0.109\end{aligned}$$

EPA guidance states that “because of the uncertainty associated with estimating the true proportion of a contaminant at a site, the 95% upper confidence limit (UCL) of the proportion should be used”. The lower bound of a confidence interval for the proportion of COPCs is not that significant. However, the upper bound of a confidence interval for the proportion of COPCs is significant. With the Systematic data for August 22, 2010, we are 95% confident that the true proportion of Uranium from area U1 containing radiation levels above 5 pCi/g, is less than 0.197. We are also 95% confident that the true proportion of Uranium from area U2 containing radiation levels above 5 pCi/g, is less than 0.109.

## 2. Confidence Intervals for the Nitrate Level in the 216-S-19 Pond

During the “screening” process of the 216-S-19 pond, Nitrate was identified as one of the COPCs. Prior to the excavation of this pond, data was collected in both areas U1 and U2 on May 8, 2010. Three different sampling techniques were used; Systematic, MIS, and Judgmental. Separate 90% confidence intervals for the mean Nitrate levels contained in each area were calculated for the Systematic data and again for the MIS data. These confidence intervals were used to address the different sampling techniques as well as the mean levels of Nitrate in the soil in areas U1 and U2. All confidence intervals were constructed using a Student’s-t distribution. The key underlying assumptions necessary to use this distribution were validated. The calculations are presented below.

**Area U1** Best point estimate for the mean Nitrate level:

**Systematic**

$$\begin{aligned}\bar{x} &= \frac{\sum x}{n} \\ &= 21.255 \text{ mg / kg}\end{aligned}$$

The margin of error constructed from 90% confidence interval for Nitrate is calculated using  $\alpha = 0.10$  significance level which is presented as follows:

$$\text{given: } Z_{\frac{\alpha}{2}} = Z_{0.10/2} = Z_{0.05} = 1.645 \text{ and: } \hat{q} = 1 - \frac{2}{39} = \frac{37}{39}$$

$$E = t_{\frac{\alpha}{2}} \left( \frac{s}{\sqrt{n}} \right)$$

$$\begin{aligned} E &= 1.685 \left( \frac{17.39901}{\sqrt{40}} \right) \\ &= 4.635477178 \\ &= 4.635 \text{ mg / kg} \end{aligned}$$

Confidence Interval of 90% for this data set:

$$\begin{aligned} \bar{x} - E &< \mu < \bar{x} + E \\ 21.255 - 4.635 &< \mu < 21.255 + 4.635 \\ 16.62 \text{ mg / kg} &< \mu < 25.89 \text{ mg / kg} \end{aligned}$$

**Area U1** Best point estimate for the mean Nitrate level:  
**MIS**

$$\begin{aligned} \bar{x} &= \frac{\sum x}{n} \\ &= 10.1 \text{ mg / kg} \end{aligned}$$

The margin of error constructed from 90% confidence interval for Nitrate is calculated using  $\alpha = 0.10$  significance level which is presented as follows:

$$\text{given: } t_{\frac{\alpha}{2}} = t_{\frac{.10}{2}} = t_{.05} = 2.132 \text{ and: } s = 0.85732 \text{ mg / kg}$$

$$E = t_{\frac{\alpha}{2}} \left( \frac{s}{\sqrt{n}} \right)$$

$$\begin{aligned} E &= 2.132 \left( \frac{0.85732}{\sqrt{5}} \right) \\ &= 0.8174198 \\ &= 0.817 \text{ mg / kg} \end{aligned}$$

Confidence Interval of 90% for this data set:

$$\begin{aligned}\bar{x} - E &< \mu < \bar{x} + E \\ 10.10 - 0.817 &< \mu < 10.10 + 0.817 \\ 9.283 \text{ mg / kg} &< \mu < 10.917 \text{ mg / kg}\end{aligned}$$

**Area U2** Best point estimate for the mean Nitrate level:

**Systematic:**

$$\begin{aligned}\bar{x} &= \frac{\sum x}{n} \\ &= 13.315 \text{ mg / kg}\end{aligned}$$

The margin of error constructed from 90% confidence interval for Nitrate is calculated using  $\alpha = 0.10$  significance level which is presented as follows:

$$\text{given: } t_{\frac{\alpha}{2}} = t_{\frac{.10}{2}} = t_{.05} = 1.685 \text{ and } : s = 13.17287 \text{ mg / kg}$$

$$\begin{aligned}E &= t_{\frac{\alpha}{2}} \left( \frac{s}{\sqrt{n}} \right) \\ E &= 1.685 \left( \frac{13.17287}{\sqrt{40}} \right) \\ &= 3.50954096 \\ &= 3.510 \text{ mg / kg}\end{aligned}$$

Confidence Interval of 90% for this data set:

$$\begin{aligned}\bar{x} - E &< \mu < \bar{x} + E \\ 13.315 - 3.510 &< \mu < 13.315 + 3.510 \\ 9.805 \text{ mg / kg} &< \mu < 16.825 \text{ mg / kg}\end{aligned}$$

**Area U2  
MIS**

Best point estimate for the mean Nitrate level:

$$\begin{aligned}\bar{x} &= \frac{\sum x}{n} \\ &= 11.82 \text{ mg / kg}\end{aligned}$$

The margin of error constructed from 90% confidence interval for Nitrate is calculated using  $\alpha = 0.10$  significance level which is presented as follows:

$$\text{given: } t_{\frac{\alpha}{2}} = t_{\frac{.10}{2}} = t_{.05} = 2.132 \text{ and } s = 3.076 \text{ mg / kg}$$

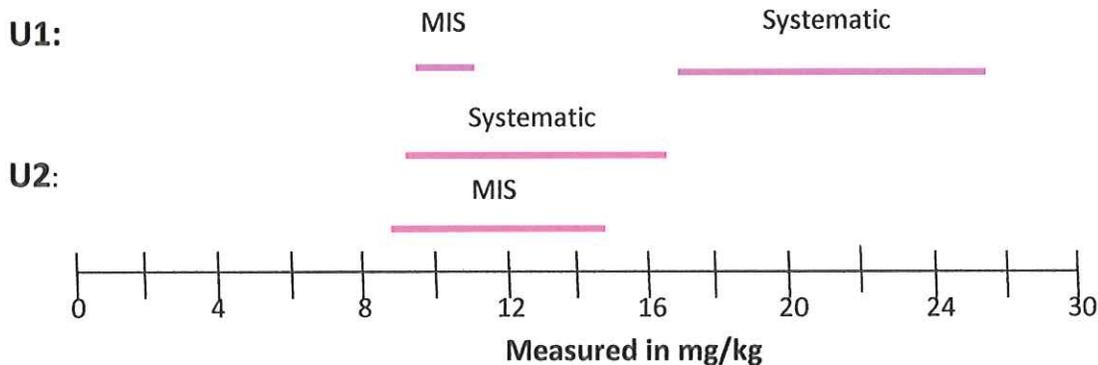
$$\begin{aligned}E &= t_{\frac{\alpha}{2}} \left( \frac{s}{\sqrt{n}} \right) \\ E &= 2.132 \left( \frac{3.076}{\sqrt{5}} \right) \\ &= 2.932875395 \\ &= 2.933 \text{ mg / kg}\end{aligned}$$

Confidence Interval of 90% for this data set:

$$\begin{aligned}\bar{x} - E &< \mu < \bar{x} + E \\ 11.82 - 2.933 &< \mu < 11.82 + 2.933 \\ 8.887 \text{ mg / kg} &< \mu < 14.753 \text{ mg / kg}\end{aligned}$$

A graphic comparison of the Nitrate levels of systematic and MIS data collected from areas U1 and U2 in May of 2010 is presented below.

**Confidence Intervals for Mean Nitrate Levels for May 8, 2010**



Area U1 appears to have a difference in the mean Nitrate levels because of how far apart the two intervals are from each other; there is no overlap here. In U2, the Systematic and MIS confidence intervals have an overlap, which shows that there doesn't appear to be any difference in the mean for Nitrate levels for that area.

Again, EPA guidance states that "because of the uncertainty associated with estimating the true average concentration of a contaminant at a site, the 95% upper confidence limit (UCL) of the arithmetic mean should be used." For the May 8, 2010 data collected in area U1 using systematic sampling, we are 95% confident that the mean Nitrate level is less than 25.89 mg/kg. For the MIS data collected during the same time period in area U1 we are 95% confident that the mean Nitrate level less than 10.917 mg/kg. For May 21, 2010 we are 95% confident that the data collected in area U2 for Systematic sampling has a mean Nitrate level less than 16.825 mg/kg. During this same time frame, we are 95% confident that the data collected in area U2 for MIS has a mean Nitrate level less than 14.753 mg/kg.

### 3. Hypothesis Tests for Nitrate Levels in the 216-S-19 Pond

According to Hanford Contamination Levels, Nitrate levels of 40 mg/kg or more are a cause for concern that could lead to area cleanup. There were no MIS samples collected on May 8, 2010 in either area U1 or U2 that exceeded this level. However, during the same time period there were 7 Systematic samples out of 40 in area U1 and 3 Systematic samples out of 40 in area U2 that exceeded this contamination level. These samples are identified in the chart below.

**Nitrate Levels for May 8, 2010 Above 40 mg/kg**

Systematic Data	ID Number	Nitrate Level
<b>Area U1</b>	B241K4	44
	B241K8	42
	B241L0	47
	B241L9	71
	B241M0	60
	B241P0	48
	B241P2	48
<b>Area U2</b>	B241R3	49
	B241R4	61
	B241W1	43

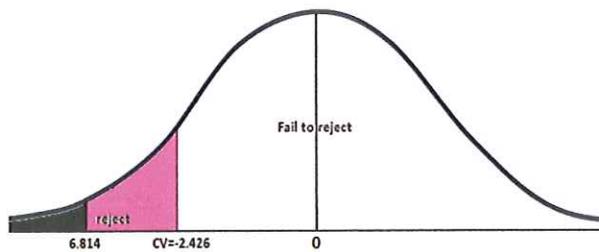
Hypothesis tests were conducted to formally test the claim that the mean Nitrate levels for the May 8, 2010 Systematic data in areas U1 and U2 are below the Hanford Contamination Level. These two tests are presented below.

### May 8, 2010 Systematic Data for Area U1:

$$\text{Given : } n = 40 \quad \bar{x} = 21.255 \text{ mg / kg} \quad s = 17.399 \text{ mg / kg} \quad \alpha = 0.01 \quad t_{\alpha/2} = t_{.01} = -2.426$$

$$H_0 : \mu = 40 \text{ mg / kg}$$

$$H_1 : \mu < 40 \text{ mg / kg} \quad (\text{Claim})$$



Student's t-distribution for Nitrate Levels in Area U1 from May 8, 2010

$$t^* = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

$$= \frac{21.255 - 40}{17.399 / \sqrt{40}}$$

$$t^* = -6.813827776$$

$$t^* = -6.814$$

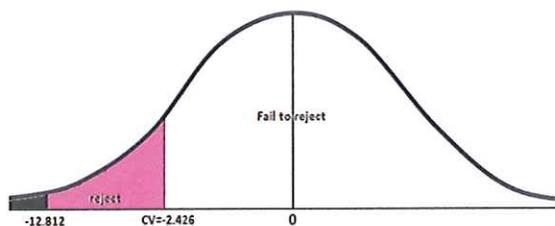
Since the test statistic of -6.814 is less than the critical value of -2.426, reject the null hypothesis. There is sufficient evidence to support the claim that the mean Nitrate level in area U1 on May 8, 2010 is less than the contamination action level of 40 mg/kg.

### May 8, 2010 Systematic Data for Area U2:

$$\text{Given : } n = 40 \quad \bar{x} = 13.315 \text{ mg / kg} \quad s = 13.173 \text{ mg / kg} \quad \alpha = 0.01 \quad t_{\alpha/2} = t_{.01} = -2.426$$

$$H_0 : \mu = 40 \text{ mg / kg}$$

$$H_1 : \mu < 40 \text{ mg / kg} \quad (\text{Claim})$$



Student's t-distribution for Nitrate levels in Area U2 Collected May 8, 2010

$$t^* = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

$$= \frac{13.315 - 40}{13.173 / \sqrt{40}}$$

$$t^* = -12.81186964$$

$$t^* = -12.812$$

Since the test statistic of -12.812 is less than the critical value of -2.426, reject the null hypothesis. There is sufficient evidence to support the claim that the mean Nitrate level in area U2 on May 8, 2010 is less than the contamination action level of 40 mg/kg.

#### 4. Calculating Sample Size

In order to meet EPA standards, the Hanford test designers determined that they want the estimates of the mean Nitrate level in areas U1 and U2 to be within 5 mg/kg of the true population mean Nitrate level. The number of usable systematic samples that need to be randomly collected from each of the areas U1 and U2 were calculated using a 95% confidence level. The population standard deviation is assumed to be 15.0 mg/kg.

$$\text{Given: } \sigma = 15.0 \text{ mg / kg } \quad E = 5 \text{ mg / kg } \quad \alpha = 0.05 \quad Z_{.05/2} = 1.96$$

$$n = \left[ \frac{Z_{\alpha/2} (\sigma)}{E} \right]^2$$
$$n = \left[ \frac{1.96(15.0)}{5} \right]^2$$
$$= 34.5744$$
$$n = 35 \text{ samples}$$

As a result of these calculations, the test planners requested 35 usable systematic samples from each of the areas U1 and U2 for both May and August of 2010 for analysis.

## Hypothesis Tests Comparing MIS, Systematic, and Judgmental Sampling

In 2008, Washington Department of Ecology in cooperation with DOE (Department of Energy) and CHPRC (CH2M Hill Plateau Remediation Company) collected two soil samples from a single location immediately adjacent to the Point of Discharge of the 216-S-19 Pond, a waste site of the 200-MG-1 Operable Unit. These samples were designed as "screening" in order to determine if the site would be a suitable location to conduct a comparison of three (Judgmental, Systematic Random, and Multi-Incremental) sampling designs. Based on the results of the Washington State Department of Ecology screening effort, COPCs (Chemical of Potential Concern) were selected and are the following: Chromium, Copper, Zinc, Mercury, Uranium-238, Uranium-233/234, Uranium-235, Plutonium-238, Plutonium-239/240, Americium-241, and Nitrate.

MIS sample points were selected by dividing each Decision Unit into grids with 100 units. One sample increment was collected from each grid unit for a total of 100 increments to comprise a single, multi-incremental "parent" sample. Four field replicate samples were also collected from each of the 100 grid-units in each Decision Unit.

Systematic Random sample points were selected using the 100-grid locations established in the MIS scheme above. Discrete sampling locations were proportioned out evenly within each Decision Unit using a random start point. In order to achieve a uniform distribution over each Decision Unit, 42 sample locations were identified rather than 40 as specified in the SAP.

Judgmental sample points were selected primarily based on field observations, professional judgment, and radiological field screening measurements. One location of highest expected (encountered) concentration will be selected, with the remaining four locations fanning out from that position. A total of five locations within each of the two Decision Units were identified and sampled.

Comparison testing of the mean concentration level of each of the elements listed above was done for these three different sampling techniques. Due to the nature of MIS sampling, the Central Limit Theorem applies, and this data can be assumed to be normally distributed as can the Systematic sampling data. The same assumption was made for the Judgmental sample data. However, any results involving Judgmental sampling should be viewed with caution as the assumption of normality is questionable. Due to time constraints, only the data collected on August 22, 2010 from area U1 were used in this analysis.

All comparison tests were conducted using a Student's t Distribution. The results for each test are summarized by element in various tables presented on the following pages. All 33 hypothesis tests used the following general format, test statistic, significance level, and critical value.

### General Approach: Testing a Claim about Two Independent Population Means

**Claim:** There is no difference, when sampling the same area, between the mean element levels obtained from MIS sampling, Systematic sampling, and Judgmental sampling.

Hypothesis Test:

$$H_o : \mu_1 - \mu_2 = 0 \text{ (Claim)}$$

$$H_1 : \mu_1 - \mu_2 \neq 0$$

Test Statistic:

$$t^* = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Significance Level:  $\alpha = 0.05$

Critical Value:  $t_{0.05/2} = \pm 2.776$

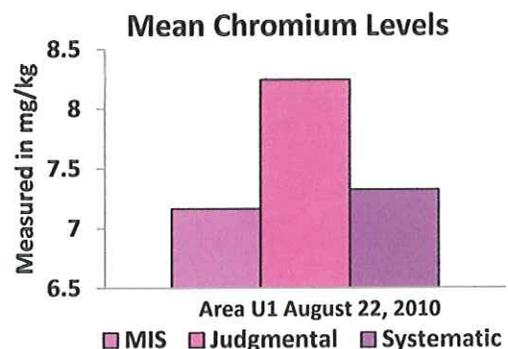
Degrees of Freedom: 4

Each element section is also accompanied by a bar graph which provides a visual comparison of the three sampling techniques and a table displaying the sample statistics used each hypothesis test. The results of these hypothesis tests are presented in a table along with the outcomes and written conclusions for each set of comparisons.

## 1. Comparison Tests Involving Chromium

### Chromium Summary Statistics in mg/kg

Sampling Method	Mean	Standard Deviation
MIS	7.16	0.384707
Judgmental	8.24	0.559464
Systematic	7.3194444	1.4089144



## Chromium Hypothesis Test Results

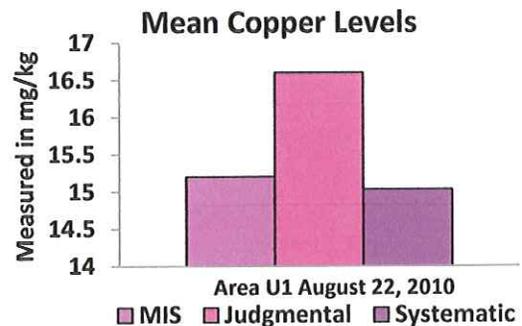
Hypothesis Test	Test Statistic	Outcome
MIS vs Systematic	-0.548	Fail to Reject $H_0$
MIS vs Judgmental	-3.557	Reject $H_0$
Systematic vs Judgmental	-2.683	Fail to Reject $H_0$

**Conclusion:** Since the test statistics for both the comparison of MIS and Systematic sampling as well as Systematic and Judgmental sampling (-0.548 and -2.683 respectively) are greater than the critical value of -2.776 and less than the critical value of 2.776, fail to reject  $H_0$ . There is insufficient evidence to reject the claim that there is no difference between mean Chromium levels obtained from MIS and Systematic sampling as well as Systematic and Judgmental sampling. However, the test statistic for the comparison of MIS and Judgmental sampling (-3.557) is less than the critical value of -2.766, reject  $H_0$ . There is sufficient evidence to reject the claim that there is no difference between Chromium levels obtained from MIS and Judgmental sampling.

## 2. Comparison Tests Involving Copper

### Copper Summary Statistics in mg/kg

Sampling Method	Mean	Standard Deviation
MIS	15.2	1.6432
Judgmental	16.6	2.6077
Systematic	15.03	1.7966



## Copper Hypothesis Test Results

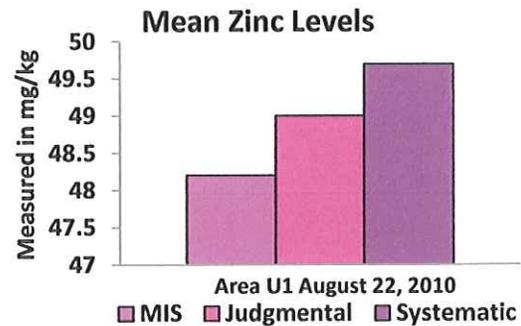
Hypothesis Test	Test Statistic	Outcome
MIS vs Systematic	0.217	Fail to Reject $H_0$
MIS vs Judgmental	-1.016	Fail to Reject $H_0$
Systematic vs Judgmental	-1.304	Fail to Reject $H_0$

**Conclusion:** Since the test statistics in all three comparisons (0.217, -1.016, and -1.009) are greater than the critical value of -2.776, and less than the critical value of 2.776 fail to reject  $H_0$ . There is insufficient evidence to reject the claim that there's no difference between mean Copper levels obtained from MIS, Systematic, and Judgmental sampling.

### 3. Comparison Tests Involving Zinc

#### Zinc Summary Statistics in mg/kg

Sampling Method	Mean	Standard Deviation
MIS	48.2	1.0954
Judgmental	49	2.3452
Systematic	49.694	2.9551



#### Zinc Hypothesis Test Results

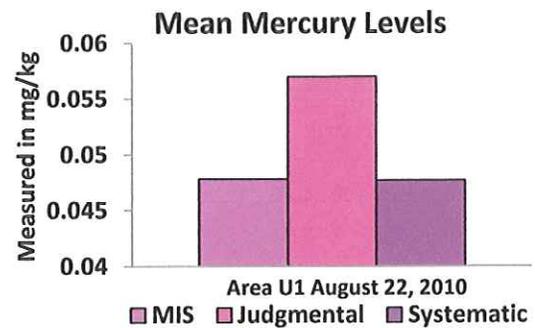
Hypothesis Test	Test Statistic	Outcome
MIS vs Systematic	-2.15	Fail to Reject $H_0$
MIS vs Judgmental	-0.691	Fail to Reject $H_0$
Systematic vs Judgmental	0.600	Fail to Reject $H_0$

**Conclusion:** Since the test statistics in all three comparisons (-2.15, -0.691, and 0.600) are greater than the critical value of -2.776, and less than the critical value of 2.776 fail to reject  $H_0$ . There is insufficient evidence to reject the claim that there's no difference between mean Zinc levels obtained from MIS, Systematic, and Judgmental sampling.

#### 4. Comparison Tests Involving Mercury

##### Mercury Summary Statistics in mg/kg

Sampling Method	Mean	Standard Deviation
MIS	0.0478	0.0060
Judgmental	0.057	0.0174
Systematic	0.0477	0.0308



##### Mercury Hypothesis Test Results

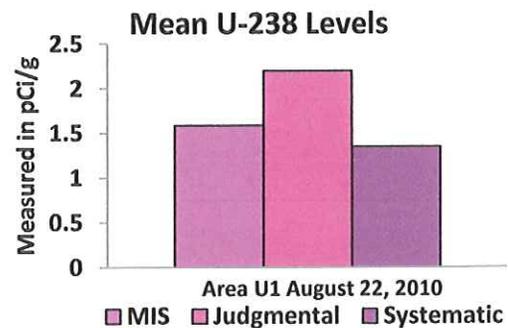
Hypothesis Test	Test Statistic	Outcome
MIS vs Systematic	0.0022	Fail to Reject $H_0$
MIS vs Judgmental	-1.118	Fail to Reject $H_0$
Systematic vs Judgmental	-0.998	Fail to Reject $H_0$

**Conclusion:** Since the test statistics in all three comparisons (0.0022, -1.118, and -0.998) are greater than the critical value of -2.776, and less than the critical value of 2.776 fail to reject  $H_0$ . There is insufficient evidence to reject the claim that there's no difference between mean Mercury levels obtained from MIS, Systematic, and Judgmental sampling.

#### 5. Comparison Tests Involving Uranium-238

##### U-238 Summary Statistics in pCi/g

Sampling Method	Mean	Standard Deviation
MIS	1.584	0.0853
Judgmental	2.196	1.5268
Systematic	1.3464	0.8110



## U-238 Hypothesis Test Results

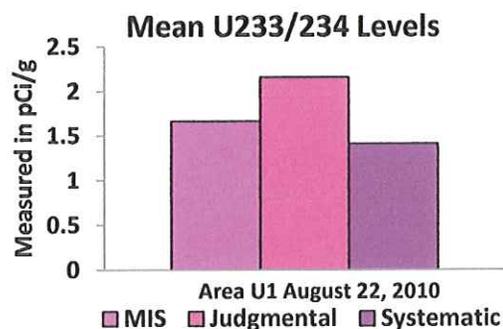
Hypothesis Test	Test Statistic	Outcome
MIS vs Systematic	1.692	Fail to Reject $H_0$
MIS vs Judgmental	-0.895	Fail to Reject $H_0$
Systematic vs Judgmental	-1.221	Fail to Reject $H_0$

**Conclusion:** Since the test statistics in all three comparisons (1.692, -0.895, and -1.221) are greater than the critical value of -2.776, and less than the critical value of 2.776 fail to reject  $H_0$ . There is insufficient evidence to reject the claim that there's no difference between mean Uranium-238 levels obtained from MIS, Systematic, and Judgmental sampling.

## 6. Comparison Tests Involving Uranium-233/234

### U-233/234 Summary Statistics in pCi/g

Sampling Method	Mean	Standard Deviation
MIS	1.662	0.0947
Judgmental	2.154	1.3202
Systematic	1.4106	0.8155



## U-233/234 Hypothesis Test Results

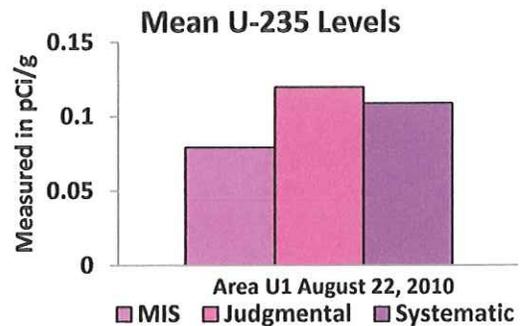
Hypothesis Test	Test Statistic	Outcome
MIS vs Systematic	1.766	Fail to Reject $H_0$
MIS vs Judgmental	-0.831	Fail to Reject $H_0$
Systematic vs Judgmental	-1.227	Fail to Reject $H_0$

**Conclusion:** Since the test statistics in all three comparisons (1.766, -0.831, and -1.227) are greater than the critical value of -2.776, and less than the critical value of 2.776 fail to reject  $H_0$ . There is insufficient evidence to reject the claim that there's no difference between mean Uranium-233/234 levels obtained from MIS, Systematic, and Judgmental sampling.

## 7. Comparison Tests Involving Uranium 235

### U-235 Summary Statistics in pCi/g

Sampling Method	Mean	Standard Deviation
MIS	0.079	0.0157
Judgmental	0.1198	0.0642
Systematic	0.1086	0.2152



### U-235 Hypothesis Test Results

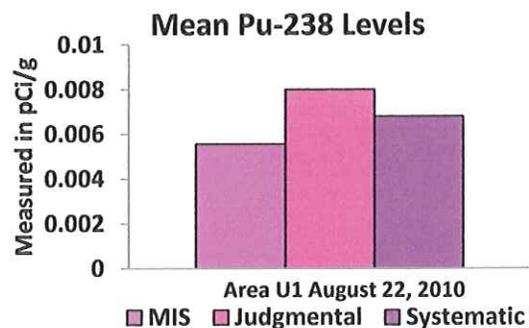
Hypothesis Test	Test Statistic	Outcome
MIS vs Systematic	-0.810	Fail to Reject $H_0$
MIS vs Judgmental	-1.380	Fail to Reject $H_0$
Systematic vs Judgmental	-0.244	Fail to Reject $H_0$

**Conclusion:** Since the test statistics in all three comparisons (-0.810, -1.380, and -0.244) are greater than the critical value of -2.776, and less than the critical value of 2.776 fail to reject  $H_0$ . There is insufficient evidence to reject the claim that there's no difference between mean Uranium-235 levels obtained from MIS, Systematic, and Judgmental sampling.

## 8. Comparison Tests Involving Plutonium-238

### Pu-238 Summary Statistics in pCi/g

Sampling Method	Mean	Standard Deviation
MIS	0.00556	0.004326
Judgmental	0.008	0.0051
Systematic	0.006794	0.004738



### Pu-238 Hypothesis Test Results

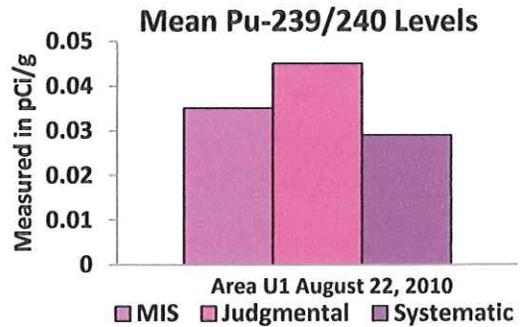
Hypothesis Test	Test Statistic	Outcome
MIS vs Systematic	-0.588	Fail to Reject $H_0$
MIS vs Judgmental	-0.816	Fail to Reject $H_0$
Systematic vs Judgmental	-0.497	Fail to Reject $H_0$

**Conclusion:** Since the test statistics in all three comparisons (-0.588, -0.816, and -0.497) are greater than the critical value of -2.776, and less than the critical value of 2.776 fail to reject  $H_0$ . There is insufficient evidence to reject the claim that there's no difference between mean Plutonium-238 levels obtained from MIS, Systematic, and Judgmental sampling.

### 9. Comparison Tests Involving Plutonium-239/240

#### Pu-239/240 Summary Statistics in pCi/g

Sampling Method	Mean	Standard Deviation
MIS	0.035	0.00815
Judgmental	0.045	0.03102
Systematic	0.029	0.03304



### Pu-239/240 Hypothesis Test Results

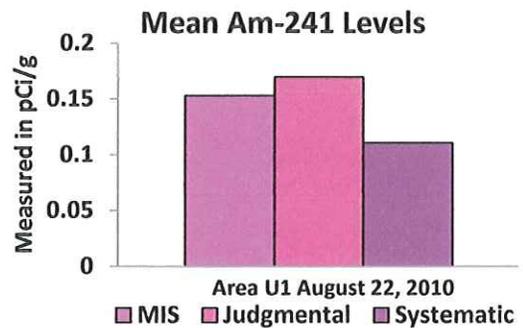
Hypothesis Test	Test Statistic	Outcome
MIS vs Systematic	0.925	Fail to Reject $H_0$
MIS vs Judgmental	-0.697	Fail to Reject $H_0$
Systematic vs Judgmental	-1.072	Fail to Reject $H_0$

**Conclusion:** Since the test statistics in all three comparisons (0.925, -0.697, and -1.072) are greater than the critical value of -2.776, and less than the critical value of 2.776 fail to reject  $H_0$ . There is insufficient evidence to reject the claim that there's no difference between mean Plutonium-239/240 levels obtained from MIS, Systematic, and Judgmental sampling.

## 10. Comparison Tests Involving Americium-241

### Am-241 Summary Statistics in pCi/g

Sampling Method	Mean	Standard Deviation
MIS	0.1528	0.02381
Judgmental	0.1696	0.111215
Systematic	0.11039	0.18687



### Am241 Hypothesis Test Results

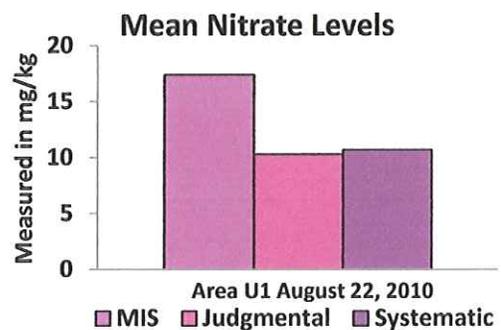
Hypothesis Test	Test Statistic	Outcome
MIS vs Systematic	1.288	Fail to Reject $H_0$
MIS vs Judgmental	-0.330	Fail to Reject $H_0$
Systematic vs Judgmental	-1.009	Fail to Reject $H_0$

**Conclusion:** Since the test statistics in all three comparisons (1.288, -0.330, and -1.009) are greater than the critical value of -2.776, and less than the critical value of 2.776 fail to reject  $H_0$ . There is insufficient evidence to reject the claim that there's no difference between mean Americium-241 levels obtained from MIS, Systematic, and Judgmental sampling.

## 11. Comparison Tests Involving Nitrate

### Nitrate Summary Statistics in mg/kg

Sampling Method	Mean	Standard Deviation
MIS	17.4	1.8166
Judgmental	10.28	11.4305
Systematic	10.692	8.9494



## Nitrate Hypothesis Test Results

Hypothesis Test	Test Statistic	Outcome
MIS vs Systematic	3.949	Reject $H_0$
MIS vs Judgmental	1.376	Fail to Reject $H_0$
Systematic vs Judgmental	0.077	Fail to Reject $H_0$

**Conclusion:** Since the test statistics for both the comparison of MIS and Judgmental sampling as well as Systematic and Judgmental sampling (1.376 and 0.077 respectively) are greater than the critical value of -2.776 and less than the critical value of 2.776, fail to reject  $H_0$ . There is insufficient evidence to reject the claim that there is no difference between mean Nitrate levels obtained from MIS and Judgmental sampling as well as Systematic and Judgmental sampling. However, the test statistic for the comparison of MIS and Systematic sampling (3.949) is less than the critical value of -2.766, reject  $H_0$ . There is sufficient evidence to reject the claim that there is no difference between Nitrate levels obtained from MIS and Systematic sampling.

**Comparison Summary:** For August 22, 2010's data collected from area U1, there were only two cases in which the null hypothesis was rejected. Chromium, being one of them, was rejected for MIS vs. Judgmental comparison samples. This case should not be taken too serious because of the fact that it fell into the Judgmental sampling method, which is already questionable. The other case that resulted in a difference where the null hypothesis was rejected was found in the MIS vs. Systematic, comparison for the element Nitrate. Being that there were multiple hypothesis tests conducted for the same period of time, it would not be totally unexpected to reject  $H_0$  when, in fact, it is true. Because there was a significance level of 0.05, it defines the probability of rejecting  $H_0$  when in fact it is true. There were 11 hypothesis tests comparing MIS and Systematic sampling means for various elements. Of those, only one detected a statistical difference. The significance level states that this mistake will be made approximately 5% of the time. This mistake is defined as finding a statistical difference when one does not exist. Further analysis should be conducted to determine whether or not this statistical difference actually exists.

**Appendix A**  
**Hanford Soil Samples Data Base**  
**Collected in May and August of 2010**

Location	Date	ID	Chromium	Copper	Zinc	Mercury	U 238	U233/234	U235	Pu238	Pu239/240	Am 241	Nitrate
U1	8/22/2010	B23NF3	7	14	48	0.047	1.62	1.73	0.085	0.0007	0.036	0.149	18
		B23NH6	7	15	47	0.058	1.71	1.68	0.07	0.01	0.027	0.152	17
		B23NH7	7.2	14	48	0.046	1.57	1.55	0.093	0.009	0.039	0.188	15
	MIS closeout	B23NH8	7.8	15	50	0.042	1.49	1.58	0.091	0.0068	0.027	0.154	20
		B23NH9	6.8	18	48	0.046	1.53	1.77	0.056	0.0013	0.046	0.121	17
	column means		7.16	15.2	48.2	0.0478	1.584	1.662	0.079	0.00556	0.035	0.1528	17.4
	clmn StDev		0.3847077	1.6432	1.095	0.00602	0.08532	0.09471	0.0157	0.00433	0.0081548	0.02381	1.81659
U1		B23NP4	8.4	20	51	0.059	1.92	2.06	0.112	0.004	0.036	0.281	9.3
		B23NP5	8.2	17	50	0.055	4.6	4.18	0.194	0.002	0.023	0.094	0.8
	Judgemental	B23NP6	7.3	14	46	0.056	1.29	1.42	0.118	0.014	0.094	0.147	5.5
		B23NP7	8.6	18	51	0.082	2.56	2.45	0.154	0.008	0.055	0.287	30
		B23NP8	8.7	14	47	0.033	0.61	0.66	0.021	0.012	0.017	0.039	5.8
	column mean		8.24	16.6	49	0.057	2.196	2.154	0.1198	0.008	0.045	0.1696	10.28
	Clmn StDev		0.559464	2.6077	2.345	0.01739	1.52677	1.32022	0.06423	0.0051	0.0310242	0.11122	11.4305
U1	Systematic	B241X6	7	15	46	0.051	1.74	2.05	0.089	0.003	0.018	0.072	6.1
		B241X7	9.9	20	55	0.044	1.33	1.5	0.11	0.003	0.035	0.047	16
		B241X8	7.4	14	47	0.036	0.64	0.74	0.06	0.008	0.034	0.113	2.5
		B241X9	7.6	13	49	0.05	0.59	0.55	0.038	0.01	0.15	1.09	2.7
		B241Y0	6.4	13	47	0.034	0.94	1.15	0.069	0.0072	0.028	0.111	15
		B241Y1	8.8	16	52	0.035	1.94	2.04	0.098	0.013	0.0008	0.009	34
		B241Y2	8.5	15	52	0.041	1.87	1.73	0.086	0.0053	0.019	0.063	35
		B241Y3	6.4	14	48	0.033	1.36	1.31	0.092	0	0.027	0.059	18
		B241Y4	9.5	19	54	0.036	1.05	1.15	0.049	0.015	0.024	0.035	34
		B241Y5	7.5	15	51	0.048	1.07	1.13	0.043	0.0028	0.068	0.164	16
		B241Y6	9.2	17	52	0.034	0.57	0.68	0.02	0.0051	0.0053	0.003	2.8
		B241Y7	6.9	14	50	0.034	1.07	1.22	0.032	0.009	0.021	0.143	14
		B241Y8	8	16	52	0.034	0.88	0.96	0.058	0.012	0.029	0.085	12
		B24200	10	17	56	0.035	0.75	0.71	0.025	0.009	0.012	0.066	19
		B24201	7.4	16	52	0.034	0.79	0.8	0.032	0	0.007	0.037	18
		B24202	6.4	13	48	0.033	1.35	1.42	0.143	0.017	0	0.042	7.2
		B24203	8.7	17	55	0.035	0.9	0.92	0.057	0.0007	0.0061	0.053	15
		B24204	8.5	16	52	0.092	0.8	0.8	0.037	0.011	0.084	0.307	12
		B24207	8.5	16	49	0.041	1.28	1.34	0.093	0.0067	0.03	0.09	4.4

B24208	9.2	17	52	0.2	3.02	2.8	1.34	0.009	0.059	0.241	5.5		
B24209	9	18	52	0.035	4.03	4.24	0.171	0.003	0.009	0.032	4.3		
B24210	7.5	15	50	0.034	3.2	3.01	0.165	0.0024	0.013	0.015	7.4		
B24211	5.6	14	50	0.033	0.62	0.69	0.026	0.0007	0.027	0.024	6.8		
B24212	6.8	14	48	0.034	0.49	0.6	0.053	0.0045	0.0068	0.006	7.9		
B24213	5.8	13	48	0.033	0.43	0.43	0.014	0.004	0.012	0.029	3.8		
B24215	6.7	13	48	0.047	1.76	1.78	0.114	0.015	0.039	0.121	7.5		
B24216	5.6	13	47	0.038	1.17	1.32	0.076	0.011	0.027	0.107	4.6		
B24217	5.8	13	47	0.033	0.69	0.66	0.052	0.005	0.003	0.021	13		
B24218	5.5	16	49	0.042	1.79	1.69	0.076	0.016	0.131	0.38	2.2		
B24219	6.2	13	50	0.033	1.36	1.28	0.049	0.0066	0.011	0.127	2		
B24220	7.8	15	50	0.098	0.74	0.77	0.052	0.001	0.011	0.035	10		
B24221	5.5	14	47	0.084	0.82	1	0.049	0.0061	0.01	0.071	3.2		
B24222	5.7	15	49	0.059	1.87	2.41	0.146	0.004	0.033	0.074	2.2		
B24224	6.1	14	46	0.055	1.79	1.85	0.078	0.0098	0.025	0.063	4.3		
B24225	4.9	14	42	0.033	1.29	1.42	0.059	0.0027	0.008	0.016	13		
B24226	7.2	14	47	0.045	2.48	2.63	0.157	0.006	0.017	0.023	3.5		
	7.3194444	15.028	49.69	0.04767	1.34639	1.410556	0.10856	0.00679	0.0288889	0.11039	10.6917		
	1.4089144	1.7966	2.955	0.03079	0.811	0.815535	0.21524	0.00474	0.0330414	0.18687	8.94943		
Location	Date	ID	Chromium	Copper	Zinc	Mercury	U 238	U233/234	U235	Pu238	Pu239/240	Am 241	Nitrate
U2	8/22/2010	B23NJ0	3.6	15	29	0.073	1.66	1.63	0.121	0.023	0.111	0.189	26
		B23NJ1	6.8	14	47	0.042	1.06	1.07	0.077	0.0076	0.07	0.159	26
		B23NJ2	6.4	14	46	0.052	1.05	1.14	0.061	0.0076	0.127	0.127	24
		B23NJ3	6.3	14	47	0.049	1.25	1.13	0.082	0.0065	0.096	0.186	20
		B23NJ4	6.1	23	42	0.043	1.2	1.13	0.044	0.0034	0.064	0.132	24
			5.84	16	42.2	0.0518	1.244	1.22	0.077	0.00962	0.0936	0.1586	24
			1.2778889	3.937	7.662	0.01256	0.24825	0.230868	0.02875	0.00767	0.0267264	0.02907	2.44949
		B23NP9	6.3	14	47	0.047	0.86	0.93	0.045	0.029	0.264	0.54	9.9
		B23NR1	7.9	15	49	0.034	1.13	1.16	0.089	0.01	0.042	0.155	45
		B23NR2	7.2	15	49	0.033	1.3	1.38	0.088	0.012	0.088	0.183	11
		B23NR3	6.8	15	46	0.034	0.8	0.91	0.071	0.01	0.024	0.037	9.7
		B23NR4	7.9	15	50	0.035	1.66	1.59	0.039	0	0.021	0.117	27
			7.22	14.8	48.2	0.0366	1.15	1.194	0.0664	0.0122	0.0878	0.2064	20.52
			0.6978539	0.4472	1.643	0.00586	0.34986	0.292797	0.02349	0.0105	0.1020745	0.1944	15.506
Location	Date	ID	Chromium	Copper	Zinc	Mercury	U 238	U233/234	U235	Pu238	Pu239/240	Am 241	Nitrate
U2	8/22/2010	B23NJ0	3.6	15	29	0.073	1.66	1.63	0.121	0.023	0.111	0.189	26
		B23NJ1	6.8	14	47	0.042	1.06	1.07	0.077	0.0076	0.07	0.159	26
		B23NJ2	6.4	14	46	0.052	1.05	1.14	0.061	0.0076	0.127	0.127	24
		B23NJ3	6.3	14	47	0.049	1.25	1.13	0.082	0.0065	0.096	0.186	20
		B23NJ4	6.1	23	42	0.043	1.2	1.13	0.044	0.0034	0.064	0.132	24
			5.84	16	42.2	0.0518	1.244	1.22	0.077	0.00962	0.0936	0.1586	24
			1.2778889	3.937	7.662	0.01256	0.24825	0.230868	0.02875	0.00767	0.0267264	0.02907	2.44949
		B23NP9	6.3	14	47	0.047	0.86	0.93	0.045	0.029	0.264	0.54	9.9
		B23NR1	7.9	15	49	0.034	1.13	1.16	0.089	0.01	0.042	0.155	45
		B23NR2	7.2	15	49	0.033	1.3	1.38	0.088	0.012	0.088	0.183	11
		B23NR3	6.8	15	46	0.034	0.8	0.91	0.071	0.01	0.024	0.037	9.7
		B23NR4	7.9	15	50	0.035	1.66	1.59	0.039	0	0.021	0.117	27
			7.22	14.8	48.2	0.0366	1.15	1.194	0.0664	0.0122	0.0878	0.2064	20.52
			0.6978539	0.4472	1.643	0.00586	0.34986	0.292797	0.02349	0.0105	0.1020745	0.1944	15.506

column mean  
clmn StDev

Location Date ID Chromium Copper Zinc Mercury U 238 U233/234 U235 Pu238 Pu239/240 Am 241 Nitrate  
U2 8/22/2010 B23NJ0 3.6 15 29 0.073 1.66 1.63 0.121 0.023 0.111 0.189 26  
B23NJ1 6.8 14 47 0.042 1.06 1.07 0.077 0.0076 0.07 0.159 26  
B23NJ2 6.4 14 46 0.052 1.05 1.14 0.061 0.0076 0.127 0.127 24  
B23NJ3 6.3 14 47 0.049 1.25 1.13 0.082 0.0065 0.096 0.186 20  
B23NJ4 6.1 23 42 0.043 1.2 1.13 0.044 0.0034 0.064 0.132 24  
5.84 16 42.2 0.0518 1.244 1.22 0.077 0.00962 0.0936 0.1586 24  
1.2778889 3.937 7.662 0.01256 0.24825 0.230868 0.02875 0.00767 0.0267264 0.02907 2.44949

Location Date ID Chromium Copper Zinc Mercury U 238 U233/234 U235 Pu238 Pu239/240 Am 241 Nitrate  
U2 8/22/2010 B23NP9 6.3 14 47 0.047 0.86 0.93 0.045 0.029 0.264 0.54 9.9  
B23NR1 7.9 15 49 0.034 1.13 1.16 0.089 0.01 0.042 0.155 45  
B23NR2 7.2 15 49 0.033 1.3 1.38 0.088 0.012 0.088 0.183 11  
B23NR3 6.8 15 46 0.034 0.8 0.91 0.071 0.01 0.024 0.037 9.7  
B23NR4 7.9 15 50 0.035 1.66 1.59 0.039 0 0.021 0.117 27  
7.22 14.8 48.2 0.0366 1.15 1.194 0.0664 0.0122 0.0878 0.2064 20.52  
0.6978539 0.4472 1.643 0.00586 0.34986 0.292797 0.02349 0.0105 0.1020745 0.1944 15.506

MIS Closeout

Judgmental  
closeout



Location	Date	ID	Chromium	Copper	Zinc	Mercury	U 238	U233/234	U235	Pu238	Pu239/240	Am 241	Nitrate
U1	5/8/2010	B23NF2	11	13	45	0.056	0.55	0.71	0.031	0	0.081	0.119	11
		B23NF7	6	7.2	27	0.08	0.77	0.98	0.03	0.028	1.19	0.331	9.7
	MIS	B23NF8	15	15	52	0.15	0.62	0.67	0.055	0.004	0.305	0.46	9.7 MIS
		B23NF9	17	21	51	0.23	0.73	0.72	0.021	0.023	0.297	0.36	9.1
		B23NH0	9.9	14	49	0.053	0.55	0.53	0.012	-0.011	0.083	0.099	11
	column mean		11.78	14.04	44.8	0.1138	0.644	0.722	0.0298	0.0088	0.3912	0.2738	10.1
	clmn StDev		4.334974	4.9303	10.31	0.07582	0.10188	0.163003	0.01605	0.0163	0.4597817	0.15803	0.85732
U1		B23NN2	11	19	39	0.15	0.85	0.64	0.037	0.026	1.04	0.5	6
		B23NN3	7.7	12	36	0.086	0.64	0.63	0.027	0.017	0.74	0.319	5.3
	Judgmental	B23NN4	77	150	60	37	6.3	6.3	0.246	2.41	57.1	123	23
		B23NN5	6.3	12	37	0.035	0.55	0.54	0.017	-0.006	0.007	0.007	38
		B23NN6	6.3	12	36	0.035	0.4	0.45	0.036	-0.002	0.016	0.042	30
		B23NN7	6.4	11	35	0.037	0.4	0.47	0.026	-0.003	0.052	0.052	12
	column mean		19.116667	36	40.5	6.22383	1.52333	1.505	0.06483	0.407	9.8258333	20.6533	19.05
	clmn StDev		28.414603	55.925	9.649	15.0772	2.34612	2.350377	0.08906	0.98135	23.163593	50.1398	13.4438
	Systematic	B241J9	7.7	11	49	0.043	0.56	0.64	0.032	-0.0013	0.018	0.01	15
		B241K1	7.6	10	46	0.036	0.62	0.55	0.024	-0.0011	0.0019	0.018	14
		B241K2	7.6	9.6	46	0.035	0.59	0.55	0.017	0.0023	0.015	0.018	22
		B241K3	7.5	8.9	45	0.039	0.47	0.54	0.011	0	0.01	0.021	19
		B241K4	23	14	50	0.14	0.6	0.65	0.017	0.0082	0.208	0.063	44
		B241K5	7.4	12	47	0.035	0.57	0.53	0.036	0.0006	0.0102	0.007	11
		B241K6	7.5	11	47	0.037	0.6	0.48	0.032	0.001	0.0121	0.01	26
		B241K7	7	9.9	47	0.036	0.56	0.46	0.027	-0.0022	0.016	0.003	3.3
		B241K8	7.7	10	46	0.044	0.63	0.64	0.042	0.008	0.198	0.125	42
		B241K9	7	8.3	45	0.036	0.53	0.57	0.048	0	0.0058	0.019	3.5
		B241L0	8.6	8.9	47	0.035	0.44	0.53	0.017	0	0.01	0.042	47
		B241L1	16	12	48	0.053	0.55	0.63	0.022	-0.0004	0.037	0.047	8.6
		B241L3	16	13	52	0.072	0.64	0.65	0.043	-0.0013	0.103	0.015	3.1
		B241L4	7.8	9.4	49	0.043	0.49	0.56	0.035	0.0062	0.014	0.008	11



B241V3	8.1	12	53	0.037	0.48	0.54	0.057	0.0052	0.014	0.044	7.5
B241V5	7.6	11	53	0.037	0.46	0.53	0.014	0.016	0.067	0.072	4.7
B241V6	8.4	11	53	0.037	0.46	0.5	0.031	0.004	0.002	0.026	4.5
B241V7	7.4	13	52	0.036	0.57	0.51	0.006	0.0039	0.006	0.022	22
B241V8	9.4	13	53	0.036	0.48	0.57	0.04	0	-0.0016	0.014	8.2
B241V9	7.4	11	52	0.036	0.46	0.49	0.026	-0.0019	0.017	0.008	12
B241W0	7.2	10	51	0.037	0.6	0.66	0.037	0.003	0.047	0.032	3.7
B241W1	8.5	13	54	0.13	0.83	0.89	0.05	0.006	0.275	0.295	43
B241W3	10	12	51	0.037	0.5	0.59	0.041	0.0087	0.087	0.083	13
B241W4	7.6	10	51	0.036	0.54	0.64	0.045	-0.0019	0.0037	0.025	12
B241W5	7.4	9.2	52	0.036	0.6	0.51	0.006	-0.002	0.025	0.022	25
B241W6	7.6	11	51	0.037	0.5	0.46	0.038	-0.0037	0.017	0.025	21
	7.9375	12.57	52.23	0.07295	0.62175	0.64825	0.03955	0.00622	0.22012	0.09988	13.315
	1.3590509	3.5448	1.641	0.13236	0.29309	0.291397	0.02555	0.01345	0.5685825	0.18044	13.1729

column mean

column StDev