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Allocating Resources in an Uncertain World: Water Management and Endangered Species

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Instream flows may be essential in protecting aquatic species that are threatened or endangered. An economic model of the optimal allocation of water under pure uncertainty is developed, with particular attention paid to the situation when a species characterized by critical depensation is affected by instream flows. We adopt the notion of pure uncertainty that Frank Knight and Daniel Ellsberg intended, which applies when decision makers cannot reduce their uncertainty to a unique probability distribution. Following recent work by Hansen, Sargent, and others, we employ robust control as a way to deal with this problem.

Key words: endangered species, robust control, uncertainty, ambiguity, critical depensation.

The Rio Grande silver minnow (*Hybognathus amarus*) is exemplary of many fish species in the arid west. Its stocks have declined markedly during the past fifty years as Rio Grande water has been increasingly used to meet agricultural and municipal demands. However, any efforts to help the species recover will be done in an environment of striking uncertainty (U.S. Fish and Wildlife Service 2007). How will society protect the Rio Grande silvery minnow and other endangered species when so little is known about them, but when the costs of some mitigating measures are so much more clear? Such costs include the politically unattractive option of denying water to agriculture and municipalities to leave more water in the river. Due to extremely limited knowledge, the management problem society faces does not involve risk, where the probability distributions of interest are well understood. The problem we consider in this article is instead one involving pure or *Knightian* uncertainty, often deemed *ambiguity* (Knight 1921; Ellsberg 1961).

Water is a natural resource often taken for granted: it frequently commands such a low per-unit price that few water users even know what price they pay. However, the twenty-first century may be the era when relative water scarcity becomes a fact of life (Brown 2003). Population growth and economic development place increasing pressure on water supplies in many arid regions, especially during drought. Idaho's Snake River below Milner Dam and New Mexico's Gila River and the Rio Grande below Elephant Butte Reservoir are examples of cases where water rights holders dry up streams and rivers completely, and it is legal to do so (Benson 2004).

Societal interest in protecting endangered species creates a natural conflict between private and public interests. In the United States, public interests in such species are represented in the Endangered Species Act. This law has been the focus of many well-known legal battles over land use rights (Brown and Shogren 1998) and conflicts over water (Benson 2004). Recent controversies (Barringer 2005) pit the Columbia River salmon runs against agricultural and municipal interests. Those who want to use the river's water argue that extraction should be allowed because it cannot be proved that reductions in current flows would affect species' survival. Because of the uncertainty in the hydrology and biological situation this claim cannot be completely ignored. Even some of the biologists who believe that there is a relationship between instream flows and species survival admit that there is a fair amount of uncertainty about the exact nature of that relationship (see Shaw 2005, pp. 264–267).

The instream flow management problem explicitly needs to account for the depth of

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the uncertainty. Below, we tackle this, building on the ambiguity and uncertainty literature (Knight 1921; Ellsberg 1961; LeRoy and Singell 1987). Ambiguity is relevant in many natural resource problems (Shaw and Woodward 2008) and for endangered species in particular. As examples, over the period from 1991 to 1999 only about 30% of all fish stocks had known population trends (National Marine Fisheries Service 2002), and even the well-studied Columbia River Basin continues to present surprises (Barringer 2005). Even less is known about the relationship between specific environmental or habitat conditions and growth.

Our dynamic model of water allocation and fishery management explicitly introduces ambiguity and the potential for ambiguity aversion by applying robust control, an approach recently advocated by Hansen and Sargent (2001; Hansen et al. 2006). Robust control has been used to examine policies in natural resource problems including water management (Roseta-Palma and Xepapadeas 2004) and extractive fisheries (Xepapadeas and Roseta-Palma 2003). In a robust control specification, choices maximize an objective function relative to the worst case scenario that the decision maker admits. Hansen and Sargent have argued that robust control is an appropriate representation of ambiguity-averse preferences as defined by Gilboa and Schmeidler (1989).

Some Relevant Economics Literature

In their classic article, Burness and Quirk (1979) demonstrated that the doctrine of prior appropriation (DPA), which is common throughout the arid western United States, generally will not allocate water efficiently. To fully explain allocation of scarce flows under the doctrine of appropriation several modelers consider the location of the source of flow and the distance from this by each agent who desires a diversion (Johnson,Gisser, and Werner 1981). This leads to a first-order difference equation that can be used to determine water quality or quantity (Weber 2001), and the spatial dimension allows game-theoretic equilibrium allocations. As markets for water in the United States become increasingly prevalent (Howitt and Hansen 2005), those that value instream flows, typically the residual claimants, are quite literally left with no flows with which to work (Ward 1987).

The economics literature of fisheries management relates mostly to commercial harvests. Reed (1979) and Clark and Kirkwood (1986) represent early contributions to the literature on optimal management under risk, looking at stock and measurement uncertainty respectively. Reed and Clarke (1990), Saphores (2003), and Sethi et al. (2005), allow for multiple sources of uncertainty and Xepapadeas and Roseta-Palma (2003), on which we will build below, consider the extractive fisheries management problem under both risk and ambiguity.

Though there have been important efforts (Ricker 1975; Johnson and Adams 1988; Jaeger and Mikesell 2002), the connection between instream flows and fisheries management is poorly developed. Tsur and Zemel (1994) study a situation in which water has consumptive value, but where excessive withdrawals can lead to extinction of a species. They find that risk in the form of a known probability distribution over the population leads to a cautious strategy that reduces the chance of extinction.

A Dynamic Model of Instream Flow Allocation

In this section we develop a different model of the problem of water management in the presence of an endangered species. First, we incorporate the existence value of a species directly into the benefit function. Second, the uncertainty we consider relates to the growth function of the species, not to uncertainty about water flows over time. Most importantly, we not only introduce risk with a known probability distribution, but also allow for ambiguity, which we assume arises from a lack of knowledge about the true dynamics of the relationship between species' growth and instream flows.

Assume that there is a single fish stock with a population size of q_t living in a river at time t. The available water supply has two possible uses: it can be used for industry or agriculture, a_t, yielding benefits such as profits from farming, $D(a_t)$, or it can be left in the river, s_t, yielding benefits from instream uses such as recreation or hydroelectricity, $W(s_t)$. To focus our attention on uncertainty in the species' dynamics rather than on randomness in flows due to weather patterns, we assume that the total flow of water is constant at the rate R, so that $a_t + s_t = R$. For notational simplicity, we

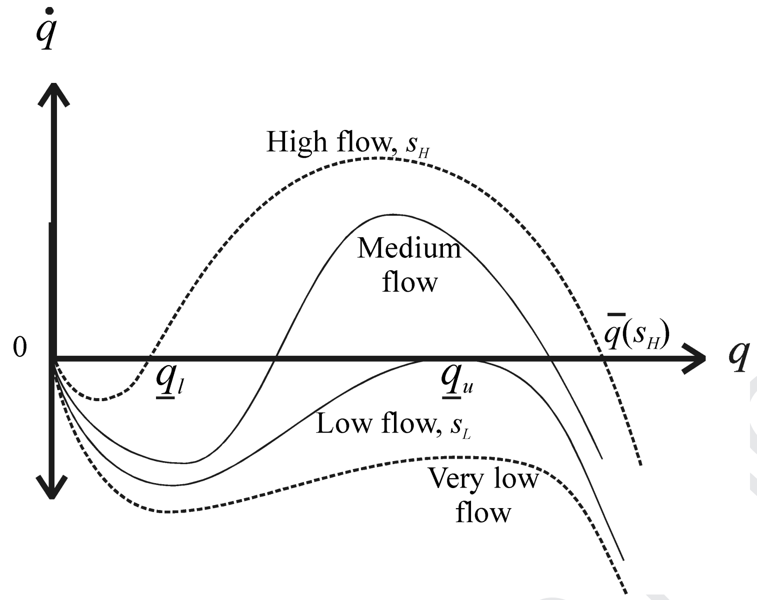

Figure 1. Biological growth with critical depensation for different instream flows

delete the time subscripts below except where necessary for exposition.

The growth of the fish stock is affected by both the current stock and the instream quantity of water, s.[1] Our main interest here is in a protected species, so harvesting is assumed to be illegal or negligible. We begin with a deterministic model in which the species' growth depends only on the current stock size and the current stream flow

$$(1) \qquad \frac{\dot{q}}{q} = f(q, s).$$

For any value of s, we assume that the dynamics of the species is characterized by a standard biological growth model with critical depensation at $\underline{q}(s)$ and carrying capacity at $\bar{q}(s)$ as in figure 1. The top two curves in the figure, labeled high and medium flow, are typical of such dynamics. At flow levels in this range, if $q < \underline{q}(s)$ then growth will be negative and, if the flow does not change, the stock will decline to extinction. If q starts above $\underline{q}(s)$ it will tend toward the carrying capacity, $\bar{q}(s)$. We assume that an increase in the instream flow improves the species' rate of growth, shifting the growth curve upward so that $\partial f(q, s)/\partial s \geq 0$.[2]

The two lower curves are logical extensions of the idea of a flow-contingent growth. In the bottom curve in figure 1, the instream flow is so low that regardless of the species' stock level, its population will decline over time. The second-lowest curve shows the growth associated with the lowest possible flow for which it is still possible to maintain a positive stock. The flow level associated with this curve, s_L, is that which would be sought by a planner seeking to maximize withdrawals for a water course (minimizing instream flows), while at the same time ensuring that the species survives. The stock that could be maintained at this flow level is $\underline{q}_u$.

For constant flows in excess of s_L, there are three steady states: stable equilibria at $q = 0$, and $q = \bar{q}$ and an unstable equilibrium at $q = \underline{q}(s)$, which lies between $\underline{q}_l$ and $\underline{q}_u$. For flows less than s_L the only equilibrium position is at $q = 0$. For $s = s_L$ there are two equilibria, at zero and $\underline{q}_u$. For some species it will be the case that for some flow levels $\underline{q}(s) = 0$; i.e., the growth function is no longer characterized

[1] One can think of instream flow in our model as a catchall for other features of habitat that affect species growth, but this strictly applies to variables that can be controlled continuously over time. Hence, for example, it would not be directly applicable to forest or other features of the landscape.

[2] Although we believe this assumption is plausible and consistent with the stylized facts, we are not aware of models that specifically include instream flows in this manner. It is possible that in some situations an increase in flow might actually diminish the rate of growth. We do not consider that case here.

by critical depensation and recovery can be achieved from any stock level. In this article we focus on cases in which $\underline{q}(s) > 0$ for all s so that there is always the possibility of negative growth.

The fish stock is assumed to have a nonuse, or passive use value, $B(q)$, sometimes deemed existence or preservation value. This nonconsumptive value might arise because the current generation wishes to preserve the species for future generations or simply because it wishes to know that the species exists (Krutilla 1967). In a model of pure existence value, the objective function would capture this value through a penalty paid if q reaches zero, as in Tsur and Zemel (1994) or Saphores (2003). We assume that $B'(q) = 0$ for $q > \underline{q}_l$; that is, the marginal value of the fish stock is zero over the range of stocks where extinction can be avoided. The cost of extinction is assumed to be finite so that there is a limited willingness to pay to prevent extinction for the decision maker. This formulation is consistent with the preferences implicit in the U.S. Endangered Species Act (ESA), which makes it illegal to place a species at risk of extinction, but imposes no restrictions once a species is delisted. Further, since the 1978 ESA amendments, the Endangered Species Committee (often referred to as the "God squad") can grant exemptions to the ESA if the Act's costs are excessive.

Optimal Water Management under Certainty

First, consider the water allocation problem with no uncertainty of any kind. The water manager's problem is to choose a level of instream flow, allocating residual water to extractive uses. This is tantamount to choosing the growth curve for the species. Assume that the manager acts as a benevolent social planner seeking to maximize the present value of net benefits obtained from the water flow over an infinite horizon subject to the constraint governing the species growth dynamics, i.e.,

$$\text{(2)} \qquad \max_s \int_{t=0}^{\infty} e^{-\rho t}(D(R-s) + W(s) + B(q))\,dt \quad s.t.\ \dot{q} = q \cdot f(q,s),$$

where ρ is the social rate of discount. The current-value Hamiltonian (J) for the problem is

$$\text{(3)} \qquad J = D(R-s) + W(s) + B(q) + \mu q f(q,s).$$

Letting f_q and f_s represent partial derivatives with respect to q and s, the first-order necessary conditions with respect to s_t and q_t are:

$$\text{(4)} \qquad D' = W' + \mu q f_s$$

$$\text{(5)} \qquad B' + \mu(q f_q + f) = \rho\mu - \dot{\mu}.$$

The second term on the right-hand side of (4) captures the value of water in protecting the species. The costate variable, μ, is a measure of the marginal value of the species; it is the stream of future benefits that arise due to a marginal increase in the stock along an optimal path. It is multiplied by qf_s, which is the impact on the growth rate of the fish stock of a marginal change in s. Hence, the term $\mu q f_s$ is the marginal value of water in terms of future benefits obtained from the fish stock.

From (4) we see that the manager's optimal allocation of water between instream and extractive uses sets the marginal value of water to agriculture, D', equal to the marginal value of the water instream including its value to instream users, W', plus its value relating to protecting the species, $\mu q f_s$. Assuming that extractive users must pay for withdrawals of water from the river, the optimal price of water, p, would reflect its complete opportunity cost,

$$\text{(6)} \qquad p = W' + \mu q f_s.$$

Of course, implementation of this rule would require the empirical analyst to overcome difficult challenges of estimation, but the conceptual goal above is quite straightforward.

Optimal Water Management under Uncertainty

We now consider the case in which the change in species stock is stochastic and there is significant uncertainty surrounding the parameters describing the dynamics of the system. We follow the steps used by Xepapadeas and Roseta-Palma (2003, hereafter XR) in their analysis of a commercial fishery, but the current model differs from XR's in a number of dimensions. First, we focus on a nonextractive fishery problem in which the species has existence value, adding the role of instream flow in the fishery's stock dynamics, characterized by critical depensation.

A stochastic optimization problem seeks a water allocation rule, $s(q, t)$, that will maximize the present value of expected net surplus, subject to the constraints of the system. Using standard procedures of stochastic control, a random variable is added to the state equation,

$$dq/_q = f(q, s)\,dt + \sigma\,dH$$

where H is a Wiener process, with $E(dH) = 0$ and $\mathrm{var}(dH) = dt$. This leads to the stochastic control problem

$$\text{(7)}\qquad \max_{\{s\}} E \int_{t=0}^{\infty} e^{-\rho t}(D(R-s) + W(s) + B(q))\,dt$$
$$s.t.\ dq_t/_q = f(q, s)\,dt + \sigma\,dH(\omega, t)$$
$$\sigma > 0, q_0 > 0 \quad \text{nonrandom}$$
$$q \geq 0, s \geq 0.$$

The term $\sigma\,dH$ is the stochastic element of the state equation; although the change in q is centered at $qf(q, s)$, it varies from that path over time with the variance increasing in a linear manner with the time horizon considered.

Despite the addition of the stochastic element in (7), the manager who solves this problem actually has quite a lot of knowledge about the system's dynamics; the parameters of the system are assumed to be known with certainty—including the moments of H. We relax this assumption and follow Hansen et al. (2006) and XR, regarding (7) as a "benchmark model." That is, we assume below that the manager does not know the parameters of the model with certainty and, given finite data available and the manager's limited ability to learn about the system through experiments, the "true" model cannot be exactly determined.

To account for this ambiguity, next define an alternative "perturbed" model that is statistically indistinguishable from (7) by setting $H = z + \int_0^t Y_\tau\,d\tau$ or $dH = dz + Y_t\,dt$, where z represents Brownian motion, and Y is drift distortion in the system's dynamics that cannot be distinguished from the standard noise. While z is symmetrically distributed around zero, the model now allows for drift over time at the rate Y. The variable Y captures the ambiguity the manager faces because of the uncertainty surrounding the actual model. The manager does not know that Y actually exists, but neither does she know that it does not exist. If there is a drift, then the state equation would be

$$\text{(8)}\qquad dq/_q = f(q, s)\,dt + \sigma(dz + Y\,dt).$$

Hence, there are two sources of uncertainty above: risk, which has a known mean and variance; and ambiguity, which reflects what the decision maker does not know.

Optimization under Ambiguity—Robust Control

Before discussing the solution of the optimization problem presented above, additional background is provided on ambiguity.[3] Since Savage (1954), most economists have assumed that decision makers act as if they have personal subjective probabilities when making choices. These probabilities are known by the decision maker, but we perhaps cannot directly observe them. The assumptions of Savage's model are more restrictive than they may first appear. Since Ellsberg's (1961) work there has been a large body of work (mostly laboratory experiments) showing that individuals often violate the Savage axioms when there are conditions of ambiguity.

In response to these violations numerous alternatives to Savage's axioms of rationality have been proposed. One approach is provided by Gilboa and Schmeidler (1989), who axiomatically show that a decision maker who faces ambiguity can rationally choose to maximize relative to the worst possible probability distribution.[4] If we accept this broader definition of economic rationality, it leads to important changes in policy rules.

In a series of articles (Hansen and Sargent 2001; Hansen et al. 2006) it has been argued that the "maxmin" optimization criterion of Gilboa and Schmeidler (1989) can be operationalized by using robust control. Robust control explicitly recognizes that there may be uncertainty in the underlying model that cannot be captured through the use of probabilities alone. It is a general form of dynamic optimization that encompasses a continuum

[3] See Kelsey and Quiggin (1992) for a good review. Shaw and Woodward (in press) discuss ambiguity in natural resource and environmental economics.

[4] A similar approach was provided by Arrow and Hurwicz (1972), whose maximin criterion is an extreme version of Gilboa and Schmeidler's criterion. Woodward and Bishop (1997) tied that criterion to the risk management concept of the *safe minimum standard*, which is the implicit decision criterion within the Endangered Species Act.

of optimization criteria, from expected net-present value at one extreme, to a maximin criterion at the other. Hansen et al. (2006) provide two possible specifications of the robust control problem, which they call the constraint and penalty problems and show that the two are mathematically equivalent. Although the constraint problem specification is most closely tied to Gilboa and Schmeidler (1989), as in XR we use the penalty problem specification in which the degree to which ambiguity affects decisions is captured through a parameter θ in the optimization problem.

Optimal Water Management under Ambiguity

The robust control specification of our instream flow problem is

$$J(q,\theta) = \max_{s_t}\min_{Y_t} E\int_{t=0}^{\infty} e^{-\rho t}\left(D(R-s_t) + W(s_t) + B(q_t) + \theta\frac{Y_t^2}{2}\right)dt \quad \text{s.t.} \tag{9}$$

$$dq/_q = f(q,s)\,dt + \sigma\,dz + \sigma Y dt. \tag{10}$$

The differences between this problem and the benchmark problem (7), are related to the robust control variable, Y. Values of Y should be "close enough to the approximating model that they are statistically difficult to distinguish from it after having observed a continuous data record of only finite length" (Hansen et al. 2006, p. 57).

There are two ways that the variable Y enters the optimization problem. First, Y enters in the state equation. Hence, by minimizing over Y, the manager behaves as if in a game against nature, in which nature is choosing Y so as to make the manager as bad off as possible. This is the maximin element of the Gilboa and Schmeidler (1989) criterion.

The second way that Y enters the robust control problem is in the objective function, through the term $\theta\frac{Y^2}{2}$. This is essentially a penalty for pessimism. As we will discuss below, the parameter θ can be thought of as the inverse of the weight placed on ambiguity aversion, which we will treat as exogenously determined. The way that ambiguity aversion enters into the solution can be understood by looking at two extremes. First, consider the case of there being no ambiguity aversion, $\theta \to +\infty$. In this case the penalty for choosing a high value of Y dominates so that the objective function is minimized at $Y = 0$. Without ambiguity aversion, therefore, the manager accepts the baseline model and ignores the possibility of drift; s is chosen to maximize the expected present value of net benefits, the standard stochastic control problem. At the other extreme, ambiguity aversion increases as θ gets smaller. If ambiguity aversion is severe, s will be chosen to maximize welfare in the worst-case setting. Intermediate values of θ, therefore, capture a continuum of degrees of aversion to ambiguity.

Following the standard approach as in XR, we next rewrite the Hamilton-Jacobi-Bellman equation for the optimization problem,

$$\rho J(q,\theta) = \max_{s}\min_{Y} D(R-s) + W(s) + B(q) + \theta\frac{Y^2}{2} + qJ_q\cdot(f(q,s)+\sigma Y) + \frac{\sigma^2}{2}qJ_{qq} \tag{11}$$

where J_q and J_{qq} are the first and second derivatives of J respectively and ρ is the discount factor as above.[5] The term J_q in equation (11) is analogous to μ in the deterministic formulation (3). Robust water extraction rules can be found by solving this optimization problem. Setting the first derivatives of equation (11) with respect to s and Y equal to zero, we obtain

$$D'(R-s^*) - W'(s^*) = J_q q f_s(q,s^*) \tag{12}$$

$$Y^* = -\frac{\sigma q J_q}{\theta}. \tag{13}$$

These two equations can be used to explore the relative roles of risk and ambiguity.

The parameter θ captures, in the words of XR (p. 12), "the maximum specification error ... that the social planner is willing to accept." We see from (13) that as θ decreases, Y^* increases in absolute value and, in the words of Whittle (2002, p. 9), a breakdown point can be reached where the decision maker is "so pessimistic that his apprehension of uncertainties completely overrides the assurance given

[5] Using Assumption 7.1 in Hansen et al. (2006), the order of the optimization criteria in (11) can be switched from max min to min max without altering the equilibrium values. As noted by a reviewer, although the equilibrium outcomes are the same, the functional form of the solutions found may differ depending on the order that the game is played out.

Table 1. Optimal Values of s, $\dot{q}$, and μ at Different Stock Levels When Growth is Possible Under the Myopic Policy ($s_M > s_L$) and When Myopic Policy Always Leads to Extinction ($s_M < s_L$)

	Range of Stock Values when $s_M > s_L$ (figure 2A)				
	$q < \underline{q}_l$	$q = \underline{q}_l$	$\underline{q}_l < q < \underline{q}(s_M)$	$q = \underline{q}(s_M)$	$\underline{q}(s_M) < q < \bar{q}(s_M)$
s	$s \approx s_M$	$s = \bar{s}$	$\bar{s} \geq s \geq s_M$	$s = s_M$	$s = s_M$
$\dot{q}$	$\dot{q} < 0$	$\dot{q} \geq 0$	*unknown*	$\dot{q} = 0$	$\dot{q} > 0$
μ	$\mu \approx 0$	$\mu \gg 0$	$\mu > 0$	$\mu = 0$	$\mu = 0$
	Range of Stock Values when $s_M < s_L$ (figure 2B)				
	$q < \underline{q}_l$	$q = \underline{q}_l$	$\underline{q}_l < q < \underline{q}(s_L)$	$q \geq \underline{q}(s_L)$	
s	$s \approx s_M$	$s = \bar{s}$	$\bar{s} \geq s$	$s < s^*$	
$\dot{q}$	$\dot{q} < 0$	$\dot{q} \geq 0$	*unknown*	$\dot{q} < 0$	
μ	$\mu \approx 0$	$\mu \gg 0$	$\mu > 0$	$0 < \mu < D'(s^*) - W'(s^*)$	

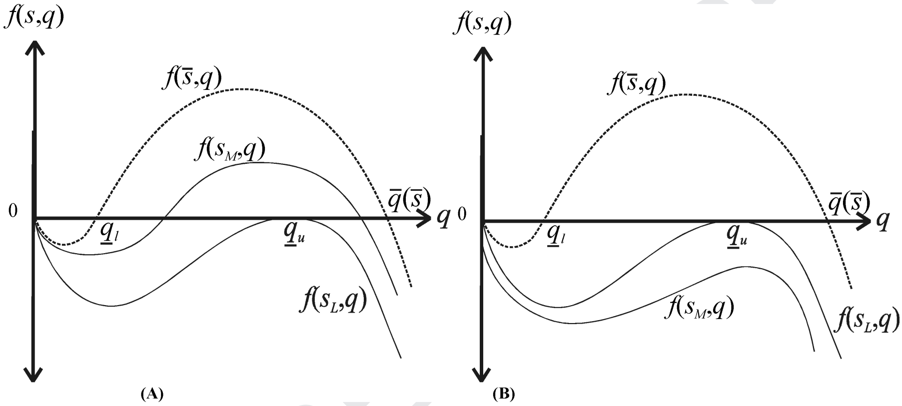

Figure 2. Biological growth function when $S_M > S_L$ (A) and when $S_M < S_L$ (B)

by known statistical behavior." XR discuss θ in much more detail. We treat it as a primitive parameter reflecting the decision maker's preferences and information, and assume that it is strictly positive and sufficiently large to avoid the breakdown point.

Optimal Policies under Certainty

The optimal policy choices under certainty, risk, and ambiguity can now be compared. Much of the intuition for this comparison can be developed by first considering the optimal policy under certainty in more detail. This policy is identified by (4), which is equivalent to (12) when $\sigma = Y = 0$. Table 1 summarizes the important variables at various parts of the state space, coinciding with the regions with different state dynamics as presented in figure 2.

Consider first the policy at $q = \underline{q}_l$, the lowest possible value for the point of critical depensation in figure 2.[6] At this point the stock sits at the precipice of extinction. If the stock falls below $\underline{q}_l$ then it will unavoidably decline to extinction. Assuming extinction is not optimal as in Clark (1973), at this point the shadow price, μ, will be high enough to induce sufficient stream flow to at least maintain the stock level. The level of s chosen at this point will be at least as high as that chosen at any other level of q. Hence, we indicate the optimal level of s at $\underline{q}_l$ as $\bar{s}$.

By definition, for $q < \underline{q}_l$, $\dot{q}$ is strictly negative and the stock will decline to extinction regardless of the level of instream flow. The marginal value of q in this range is positive only because the time of extinction is deferred somewhat. If the decline is sufficiently fast,

[6] Note, we assume that $\underline{q}_l > 0$, i.e., that even at the highest flow levels the stock is characterized by critical depensation. If $\underline{q}_l = 0$, the analysis would change little for $q > \underline{q}_l$.

over this range μ will tend to be small and the manager will choose an s that is quite close to s_M, the level of instream flow that would be chosen if the manager were myopically focused only on use-oriented benefit, i.e., where $D'(s_M) - W'(s_M) = 0$.

Next consider the upper end of the stock levels. Recall from figure 1 that s_L is the lowest flow level for which nonnegative growth is possible. If $s_M > s_L$ (figure 2A) then even without regard for the species, a flow level will be chosen that allows for positive growth. In this case any increment to the stock above $\underline{q}(s_M)$ will not cause a change in s, nor will it have any impact on future existence values experienced. Hence, if $s_M > s_L$ the marginal value of the stock will be zero for all $q \geq \underline{q}(s_M)$. If q starts at $\underline{q}(s_M)$, then growth is zero so the stock will remain at that unstable equilibrium indefinitely. If q starts above $\underline{q}(s_M)$, the species population will converge to $\bar{q}(s_M)$.

A more interesting situation occurs if $s_M < s_L$ (figure 2B). In this case the myopic choice would lead to extinction. The optimal policy will, therefore, balance the desire to increase extractions with the need to avoid collapse of the species to extinction. In this case the marginal value of species will be positive over the entire domain since an increase in the stock allows the manager to decrease s closer to s_M, at least for a time, before inevitably having to sacrifice extractive value for the sake of the species. Assuming that extinction is not optimal, we know that the long-run equilibrium will be reached at some flow level, say s^*, and a stock level of either $\underline{q}(s^*)$ or $\bar{q}(s^*)$. It turns out, however, that $\bar{q}(s^*)$ is not optimal since the sustainable path with $s = s^*$ and $q = \bar{q}(s^*)$ is dominated by a path in which $s < s^*$ for a finite period, followed by a sustainable path in which $s = s^*$ and $q = \underline{q}(s^*)$. This alternative path leads to a higher present value of net benefits and an equilibrium at the biologically unstable point of critical depensation. In the Appendix, we show that such a point can be locally economically stable because it is possible to instantaneously adjust the flow levels to push the stock back to some point $\underline{q}(s)$.

Deviations of s from s_M arise due to the marginal value of the stock, μ in equation (4) or J_q in equation (12). A stylized representation of how this parameter changes over the state space is presented in figure 3. The solid line there, which we refer to as J_q^C, represents the marginal value of the species under certainty.

Optimal Policies under Risk and Ambiguity

Equation (12) presents the first-order condition with respect to s, showing that optimal instream flow is set where the immediate marginal net benefit of extraction, $D'(\cdot) - W'(\cdot)$, is equal to the marginal cost in terms of foregone future benefits obtained from the species, $J_q q f_s$. The term $J_q q f_s$ is the marginal user cost. The effect of risk and ambiguity on optimal instream flows is captured in Jq; as long as the functions D and W are monotonic and concave, an increase in J_q will lead to increased instream flows.

Recall that here the species is exclusively valued for its existence. In this model, therefore, its direct marginal value is positive only

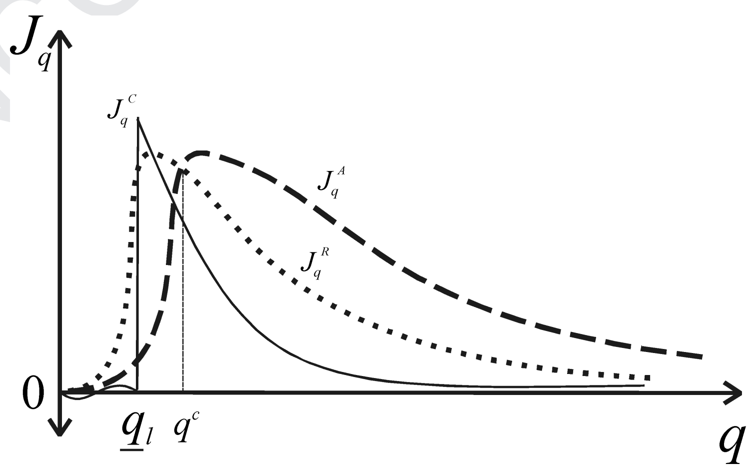

Figure 3. Marginal value of the fish stock under certainty, J_q^C, under risk, J_q^R, and with ambiguity aversion, J_q^A, assuming $s_M < s_L$

as q approaches zero. For $q > \underline{q}_l$, an increase in q is valuable because it lowers the probability that the stock size will move toward zero, where the optimal policy is to make costly increases in s.

Strictly speaking, the stock will never reach zero within a finite horizon because of the assumption of geometric growth. To simplify our discussion, we will refer to a species as being extinct if it falls below some small stock size in the neighborhood of, but is not exactly at, zero (e.g., a single individual). Because the stochastic process is a Weiner process, there is a nonzero probability that in any finite period the species will drift into the range of critical depensation from which it will, on average, tend toward zero. Over an infinite horizon, therefore, extinction is unavoidable. Over any finite horizon, however, even a very long one, the chance of actually going extinct can be near zero or near one. For any stock greater than $\underline{q}_l$ it is possible to allocate enough water to instream flow that the probability of an immediate increase in the stock is greater than 50%. Hence, if the stock starts near $\underline{q}_u$ and if from $\underline{q}_u$ to $\underline{q}_l$ the optimal policy at every point stock leads to a better than 50% chance of the stock increasing, then the probability of reaching $\underline{q}_l$ in the foreseeable future would be very small.

The effects of risk and ambiguity on J_q are presented in figure 3. The deterministic case discussed above, J_q^C, is indicated by the solid line. The case of risk without ambiguity aversion, $\sigma > 0$ and $\theta \rightarrow \infty$, is represented by the dotted line in figure 3 and will be referred to as J_q^R. The introduction of risk means that for some values of $q < \underline{q}_l$ the marginal value of s is positive as it increases the probability that the stock will grow in the immediate future. As a result, the J_q^R curve slopes upward from zero in figure 3 before $\underline{q}_l$. To the right of $\underline{q}_l$, it is possible for the average rate of growth to be positive. Risk means that there is never a range in which the stock is completely "safe" from extinction, but the greater the stock, the less likely it is that extinction will occur in any finite time period. Because risk of extinction in the near future declines gradually as q increases, J_q^R will descend more slowly than J_q^C. From (12), if follows that at any stock level where $J_q^R > J_q^C$, the flow s will be greater under risk than under certainty.

Next, consider the introduction of ambiguity aversion by reducing the penalty parameter, θ (which again, is the inverse of the weight placed on it). Ambiguity aversion changes the nature and solution to the problem in two important ways. Under ambiguity aversion the manager will behave as though nature is playing against her by choosing Y_t following (13). Because $J_q \geq 0$ across the entire domain, the minimizing value of Y will be negative, meaning that the manager will optimize relative to a growth function that is being pulled down compared to the baseline model. This leads to the J_q^A curve in figure 3, the dashed line, which is a multiplicative transformation of J_q^R, so that J_q^A is "stretched" to the right.

Because of the assumption of critical depensation with $\underline{q}_l > 0$, all the J_q curves are nonmonotonic. This means that there is a point (indicated q^c in the figure) at which J_q^A crosses J_q^R. To the right of q^c, J_q^A lies above J_q^R, meaning that ambiguity aversion leads managers to place more weight on the value of the species and allocate a greater share of the available water to instream flows. In this range, ambiguity aversion makes it optimal to be extra cautious; even when the population appears (on average) to be secure, ambiguity causes the manager to allocate more water to species protection. Hence, to the right of q^c (and assuming that the baseline model is true) the probability of the stock falling to extinction in any finite horizon is less under ambiguity aversion than it is under risk.

Figure 3, also shows however, that ambiguity aversion has a very different effect when the species population is below q^c. In this range ambiguity aversion actually reduces the marginal value given to the species in (12). The intuition behind this result is that for low stocks, the pessimistic manager will view the prospects for the species as increasingly bleak and will tend to reallocate water to the extractive use that generates tangible benefits. As a result, for the lowest and most vulnerable stocks, ambiguity aversion in our model actually leads to a reduction in instream flows, decreasing the mean rate of growth. For a given initial stock below q^c, it is possible that the policy under risk might have a better than 50% chance of avoiding extinction in the near future, while the ambiguity averse policy might tend to push the species toward a quick extinction.

Discussion of Results

Our models of optimal instream flow management under certainty, risk, and ambiguity provide a number of insights into the kinds of trade-offs that are made in making choices associated with species habitat. Several of these results are intuitively obvious: optimal management will balance net marginal benefits to extractive use with the marginal cost in terms of species loss; species management becomes smoother when risk is introduced, and ambiguity can lead to even more cautious policies. However, one key result is more difficult to understand. As shown in figure 3, there is a range of stocks where the ambiguity averse strategy is less cautious in terms of protecting the species and seem to give up on the species. What is driving this counterintuitive result that our formulation offers?

First, it may be that ambiguity as defined here is still too restrictive a notion. Quiggin (2005) argues that the precautionary principle, which has become an important heuristic for environmental planning, can be explained by reference to the incompleteness hypothesis, which allows for uncertainty that is even more broadly defined than it is here. Second, in the model specification there is a lower bound on the point of critical depensation, $\underline{q}_l$; if the stock falls below this point there is nothing that can be done to avoid extinction. Although this seems biologically reasonable, we know of no real species management problem in which managers have consciously given up all hope of species recovery. If $\underline{q}_l = 0$, then J_q^A would dominate J_q^R across the entire domain so that ambiguity would always lead to higher levels of instream flow.

Third, recall that the robust control solution follows from an assumption of ambiguity aversion. Admittedly, the degree of ambiguity aversion for any decision maker, just like risk aversion, is ultimately an empirical question. Nevertheless, to the right of q^c, the robust-control policy reduces ambiguity by pushing the stock upward, to the region where ambiguity becomes irrelevant to the manager. Near $\underline{q}_l$, on the other hand, the ambiguity surrounding the species' survival becomes most salient to the manager—a point is reached where a reduction in *s* will reduce ambiguity, which would be perceived as a benefit to the ambiguity averse decision maker. For the manager who is ambiguity averse a point is reached where focus is shifted back to foregone benefits of water's extractive use. In other words, ambiguity aversion does not mean "preserve at any cost." Instead, the maximin strategy can move a manager from making aggressive efforts to protect the species to giving up on the species as the chance of survival falls.

A final interpretation of these results might be found in a critique of robust control itself. We have proposed that policy makers' preferences might be consistent with the axioms of Gilboa and Schmeidler (1989) and that they might exhibit aversion to ambiguity. However, for very low species stocks, when extinction is most imminent, these assumptions lead to choices that seem inconsistent with what we see in the real world. Real world manager or policy-maker actions may be "wrong," or they do not adhere to these axioms, or they are not averse to ambiguity.

Nevertheless, we believe our analysis actually highlight the merits of the use of robust control. Robust control is derived from strong normative foundations based on optimal decision-making under ambiguity. Robust control is not simply a rationalization of policies that are preordained as "conservative." This echoes the findings of Giannoni (2002) who, in the context of monetary policy, identified policies that were inconsistent with "conventional wisdom."

Finally, we emphasize that the robust optimal policies do not make extinction more likely. On the contrary, to the right of q^c ambiguity increases the marginal value placed on the species, resulting in optimal polices that allocate more water to instream flows. The robust policy will, therefore, be proactive to the threats to the species and reduce the possibility that species will become endangered.

Summary and Conclusions

The connection between endangered species, water allocation, and laws such as the ESA are evident today (Benson 2004). Controversy runs high, with some arguing for compensation of damages from forgoing water use, and concerns that this will ultimately weaken the ESA (Boxall 2004), and that the burden of proof will fall to those arguing that instream flows are necessary for species protection. As an example, consider again what we know about the silvery minnow. First, Berrens, Ganderton, and Silva (1996) estimate that New Mexico households would be willing to pay between \$28 and \$90 per year to restore instream flows for the silvery minnow, a fairly wide range in values. Second, Ward and Booker (2006) find that augmenting Rio Grande instream flows sufficiently to protect the silvery minnow would actually increase overall net benefits by moving water from lower to higher-valued users. Third, however, the U.S. Fish & Wildlife Service (2007) estimates that the undiscounted cost to restore the silvery minnow's population would exceed $114 billion over twenty-five years.[7] Because of the enormous uncertainty surrounding the population dynamics of many endangered species such as this, we may be never be certain whether a particular plan of action is either necessary or sufficient to avoid the minnow's extinction. It remains to be seen whether society will allocate enough resources in time to actually save several species.

Recognition of the importance of ambiguity and the development of appropriate frameworks for decision making in light of it is an important challenge to economists today. Here we have considered the problem of water management where an endangered species exists, but where there is significant ambiguity about the species' dynamics. We find that when ambiguity aversion is present, the ambiguity-averse strategy will change policy relative to the policy under risk. For some levels of the fish stock, the potential for extinction will be given more weight in the water allocation decision, even if current populations appear to be fairly safe from extinction. On the other hand, we find that if the stock reaches a point where extinction becomes likely, then ambiguity-averse decision makers might prefer to shift resources to alternative economic activities. At least in this context, the "optimal" policy is not necessarily to try to save the species regardless of the cost.

We have left unanswered the critical question of how ambiguity should be treated by a benevolent social agent or manager. Individual decision makers regularly demonstrate ambiguity aversion. Should policy decisions also be based on ambiguity aversion, reflecting an assumed ambiguity aversion of the public? Or should policy instead be based on cold probabilities and the best available science? These are good questions to ask. There is already evidence of ambiguity aversion in public policy; the ESA requires that protection programs be robust to the uncertainty surrounding the species' survival and one could argue, therefore, that the U.S. Congress revealed a strong aversion to ambiguity when overwhelmingly approving the ESA at its inception. Ambiguity aversion provides a justification for placing weight on species preservation in water allocation problems, even in the absence of conclusive evidence of a relationship between the species and instream flow. It does not necessarily warrant extreme behavior, but it does suggest caution can be "optimal."

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[7] By our counting, two-thirds of the costs itemized are for activities that would address information gaps regarding the species and only 8% of the total estimated budget for the species' recovery would go toward activities that would increase the flow.

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Appendix

Stability of Equilibrium in Deterministic Model

The equations of motion of the deterministic system are in equations (1) and (5). Noting that s is implicitly a function of q and μ, we obtain a linear approximation of these equations around equilibrium, q_0, μ_0 from the first-order condition, equation (4)

$$\begin{aligned}\dot{\mu} \approx\ & \mu_0(\rho - q f_q(q_o, s(q_0, \mu_0))) - B'(q_0) \\ & + \left[(\rho - f_q(q, s)) - \mu\left(-f_{qs}\frac{\partial s}{\partial \mu}\right)\right](\mu - \mu_0) \\ & + \left[\mu\left(f_{qq}(q, s) + f_{qs}\frac{\partial s}{\partial q}\right) - B''\right](q - q_0),\end{aligned}$$

$$\begin{aligned}\dot{q}_t \approx q_0\Bigg(& f(q_0, s(q_0, \mu_0)) + \left[f_q + f_s\frac{\partial s}{\partial q}\right](q - q_0) \\ & + f_s\frac{\partial s}{\partial \mu}(\mu - \mu_0)\Bigg).\end{aligned}$$

The qualitative properties of this system are equivalent to the properties of the associated homogeneous system, which can be written in matrix notation,

$$\begin{bmatrix}\dot{\mu} \\ \dot{q}\end{bmatrix} \approx \begin{bmatrix}[(\rho - f_q) + \mu(f_{qs}s_\mu)] & [\mu(f_{qq} + f_{qs}s_q) - B''] \\ q_0 f_s s_\mu & q_0[f_q + f_s s_q]\end{bmatrix} \times \begin{bmatrix}(\mu - \mu_0) \\ (q - q_0)\end{bmatrix}$$

The Eigen values for this linear system of equations[8] are

$$\text{(A.1)} \quad \frac{1}{2}(f_q + \rho + q_0 f_s s_q + \mu f_{qs} s_\mu + q_0 f_q)$$

$$\pm \frac{1}{2}\begin{bmatrix}(\mu f_{qs} s_\mu)^2 + f_q^2 + (q_0 f_s s_q)^2 \\ + 2q_0(-\rho f_q - \rho f_s s_q + f_q f_s s_q + f_q^2 \\ \quad -\mu f_{qs} s_\mu f_q + \mu f_{qs} s_\mu f_s s_q + q_0 f_s s_q f_q) \\ - 2 f_q \mu f_{qs} s_{mu} + 2\rho\mu f_{qs} s_\mu - 2 f_q \rho + \rho^2 \\ \quad + q_0^2 f_q^2 + 4q_0 f_s s_\mu \mu f_{qq} - 4q_0 f_s s_\mu B_{pp})\end{bmatrix}^{1/2}$$

A necessary and sufficient condition for stability of the equilibrium is that the Eigen values have negative real parts. This can only occur if the sum of the first five terms in parentheses in (A.1) is less than zero. That is, stability will be achieved if

$$\text{(A.2)} \quad f_q + \rho + q_0 f_s s_q + \mu f_{qs} s_\mu + q_0 f_q < 0.$$

Economic intuition for the conditions in which this is satisfied can be obtained by first considering the case where $f_{qs} = 0$. At a point of critical depensation $f_q > 0$, so we can sign the various parts of this inequality as follows:

$$\text{(A.3)} \quad \underset{+}{f_q}\,(1 + \underset{+}{q_0}) + \underset{+}{\rho} + \underset{+}{q_0}\ \underset{+}{f_s}\ \underset{-}{s_q} < 0.$$

If q_0 is *large*, then $(1 + q_0) \approx q_0$ and $\frac{1}{q_0} \approx 0$. So (A.3) will hold approximately if $f_q < -f_s s_q$. The left-hand side of this inequality is the rate at which growth increases as the stock increases. The right-hand side is the rate at which growth declines due to the decrease in the instream flows as a result of the marginal increase in q. If, at the margin, the effect on the growth rate caused by the change in s outweighs the effect on the growth rate caused by the increase in the stock, then a point of critical depensation can be economically stable.

Stability is much more easily satisfied at a point of flow-contingent carrying capacity, for in this case $f_q < 0$ and the first term in (A.3) will also be negative. Using the approximating assumptions, the inequality will always hold.

Note that if $s_M > s_L$, then at the point of critical depensation, $\underline{q}(s_M)$ a marginal increase in the stock will not induce a change in s. In this case, therefore, $s_q = 0$ so that equation (A.3) cannot be satisfied so that it is quite unlikely that $\underline{q}(s_M)$ would be a stable equilibrium. In this case the carrying capacity associated with s_M, $\bar{q}(s_M)$, would be the only stable equilibrium in addition to the extinction.

When $f_{qs} \neq 0$, the relationship becomes more complicated, but the essential economic intuition remains. From equation (4), it follows that an increase in μ would cause an optimal increase in the water allocated to instream purposes if $D'' < 0$. Hence both s_μ and μ are greater than zero, so that if $f_{qs} < 0$ it is easier to satisfy (A.2) (or harder if $f_{qs} > 0$).

[8] Eigen values were calculated using Matlab.

Queries

Q1 Author: Camerer and Weber 1992 is not cited in the text. Please check.
Q2 Author: Cowley 2002 is not cited in the text. Please check.
Q3 Author: Please update the in press reference "Shaw and Woodward 2008."